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A FEASIBILITY STUDY OF A RADIATION ANALYZER FOR THE ZODIACAL LIGHT

By

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and

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PREFACE

Although the zodiacal light has been known for millenia, accurate photometric measurements of it have been attempted only recently. Because of difficulties with scattered light, airglow, and atmospheric extinction, ground based studies of surface brightness and polarization show enormous discordances. It would appear that measurements suitable for even the most rudimentary theoretical analysis would have to be made above the earth's atmosphere and in this report we have examined the feasibility of such studies. We require determinations of the surface brightness, B, and of the plane polarized and elliptically polarized components of the scattered radiation as a function of elongation ε, from the sun and celestial latitude β and of the wavelength of the radiation.

Since the surface brightness of the zodiacal light varies from 10^{-9}, that of the sun in the outer corona to 10^{-13}, that of the sun at large elongation, accurate photometry and polarimetry impose some exacting conditions. A detailed assessment of the problems involved show that with available optical materials, detectors and electronic devices it is possible to measure with satisfactory precision and at several wavelengths the intensity, the percentage polarization, the orientation of the plane of polarization and the amount of circularly polarized and unpolarized light. If one wishes, the data can be given in terms of Stokes parameters although we have not done so.

From such data, obtained at several wavelengths with a band pass of about 100 A° it should be possible to determine the spacial distribution of the particles (from measurements made over a long range in elongation) and something about the size distribution from the distribution of brightness near the sun, from the color and from the polarization. From measurements of B, and both plane and elliptical components of polarization it should be possible to infer whether the particles are dielectric or metallic and to estimate the relative properties of each and of free electrons.

In this study of the feasibility of the experiment, the main problems of: scanning procedures, light energy levels, star crossing noise, light transducer requirements, method of measurement of degree of polarization and ellipticity factors, related instrumentaion and fail-safe actuation have been considered. The feasibility has been established for the spectral range of λ3500 to λ6000 within which a large signal to noise ratio of some 50/1 or more is available leading to classical electronics treatment.

The study is presented in the ten following chapters dealing with the different aspects of the problem. It has been deemed necessary to incorporate in this work the reviewing of some well known fundamentals to avoid, later on, possible misinterpretation of astrophysical data at the designing and preliminary experimental stages. In some instances alternative solutions to problems have been suggested. It is hoped that specific knowledge of the limitations imposed by the O.S.O. vehicle will be forthcoming so as to bring the design of the proposed instrument to the engineering stage.

We are indebted to the authors of our main sources of reference. These have been:

Weinberg, I. L. - Photoelectric Polarimetry of the Zodiacal Light at λ5300 (Haleakala) - Ph.D. thesis, 1963

Blackwell, D. E. and Ingham, M. F. - Observations of the Zodiacal Light From a Very High Altitude Station - The Average Zodiacal Light - Mon. Not. Roy. Astron. Soc. V 122, p 113, 1961

Allen, C. W. - Astrophysical Quantities - London, Athlone Press, p 116, 1955

Johnson, F. S. - The Solar Constant - J. Meteor. 11, 431, 1954

Because some of the technical problems in this work are common to those encountered previously in this laboratory we have included in the present report a copy of the recent publication, "Measurement of Optical Activity: New Approaches". Science 142, 200-208 (1963).

The report is in the nature of an uncorrected page-proof. There are the usual errors caused by the typist; alas not many typists are too knowing of astrophysics and algebraic notation. Suggestions and criticism concerning the substance of this progress report will be appreciated.

A. L. Rouy

L. H. Aller

B. Carroll

CONTENTS

I

THE SCANNING OF THE ZODIACAL LIGHT FROM AN O.S.O. VEHICLE

An O.S.O. vehicle circling around the earth, above its atmosphere offers the opportunity to measure the relative brightness of the zodiacal light, its degree of polarization and similarly its ellipticity factor. Since its orbiting distance to the surface of the earth exceeds some 150 kms, the vehicle will be always higher than the "emitting layer" at an altitude, from ground, of some 100 kms and a fortiori above the ozone layer and the scattering atmosphere. These experimental conditions should eliminate or at least minimize to negligible quantities the aberring terms which limit the precision of the ground observations. The irradiant energy of the zodiacal light will be higher and its degree of polarigation as well as its ellipticity factor made more accessible without or with very little correction.

Naturally, the spinning of the vehicle about a preferred axis introduces experimental conditions completely different from the ones encountered at ground stations.

At first, it seems that such vehicle might not be suitable to conduct an orderly scanning of the zodiacal light. It may preclude, in part or in totality, the observation during a certain part of its circling cycle, mainly for that part of its trajectory behind the earth which respect to the position of the sun.

But, as a counterpart of the restriction inherent to its nature, the O.S.O. vehicle may give access to direct observation of those very same factors found integrated in the measurement of the brightness of the zodiacal light. Those factors pertaining to the emitting layer, the ozone layer and the scattering atmosphere represent a rather important percentage of correction in the evaluation of the relative brightness of the zodiacal light as obtained from a ground station. The magnitude of the energies involved would be certainly accessible to an instrumentation capable of recording a relative brightness $B/\overline{B_{\odot}}$ of the order of 10^{-13}. Yet the conditions of measurement would be quite different since they would proceed along directions of observations taugential or mearly tangential to their spherical distribution. The relative inportance of such possible measurements is not the object of this study. Yet it seems logical to mention this aspect which in all eventuality will appear as a signal pulse of short duration in the cylic scanning of the zodiacal light.

The mechanism and procedure of the scanning imposed by the nature of the O.S.O. vehicle can be outlined and discussed readily. For this purpose one can elect a set of three axes of coordinates in which the axis OZ, orthogonal to two respectively orthogonal axes OX and OY within the plane of the ecliptic, is orthozonal to the same plane of the ecliptic. The three axes pass through the sun as origin and the axis OX may be selected to coincide with the apsides line for further reference with celestial coordinates. On account of the relative distances and dimensions involved, the position of the O.S.O. vehicle can be considered as being defined by the position of the earth on its orbit in the plane of the ecliptic containing the OY and OX axes. The use of the plane of the ecliptic as plane of reference appears permissible for the general discussion. Indeed, the observations made by Blackwell [1960] and then by Weinberg [1962] indicate that the observable plane of symmetry of the zodiacal light is inclined by some $1^\circ 6'$ with respect to the plane of the ecliptic.

For the purpose of general discussion it is considered that the line of sight can be represented, at any instant, by a vector VT intersecting the axis of spin VS and making with it the angle φ . Being in fixed relationship with respect to each other. The two intersecting directions VT and VS define a plane which rotates in space around axis of spin and at the angular velocity ω as of the rotation of the vehicle.

The possibilities of the scanning and its modalitics can be defined readily by means of a space triangulation as shown. Indeed, definite angular relationships, describing fully the scanning in a general form, can be established immediately. For that purpose it suffices to consider the plane AT_1T_2 orthogonal to the axis of spin VS which also contains the center O of the sun. It is defined by its trace OT_1T_2, in the plane of the ecliptic, normal to the projection VH of the axis of spin onto it. Its uniqueness leads to the relationships:

$$VH = \frac{VA}{\cos \lambda_0} \qquad \text{I-1}$$

$$AT_1 = AT_2 = VA \tan \varphi \qquad \text{I-2}$$

$$HT_1 = AT_1 \cos \omega t_1 \qquad \text{I-3}$$

$$HT_2 = AT_2 \cos \omega t_2 \qquad \text{I-4}$$

FIGURE I-1

TRIANGULATION OF THE SCANNING OF THE ZODIACAL LIGHT

The three orthogonal vectors ox, oy, oz define the system of axes of reference whose origin coincides with the sun; the axes ox and oy being in the plane of the ecliptic.

In this system of coordinates:

OV represents the radius issued from the sun O and passing through the position of the vehicle V in the plane of the ecliptic.

α the angular distance of ov from the axis ox.

VS the axis of spin of the vehicle

λ_o the angular elevation of vs with respect to the ecliptic.

VH the projection of the axis of spin vs into the plane xoy

ε_o the elongation of the projection vh with respect to the direction vo

VT_1, VT_2 lines of sight within the plane of the ecliptic and making the angle φ with the axis of spin vs

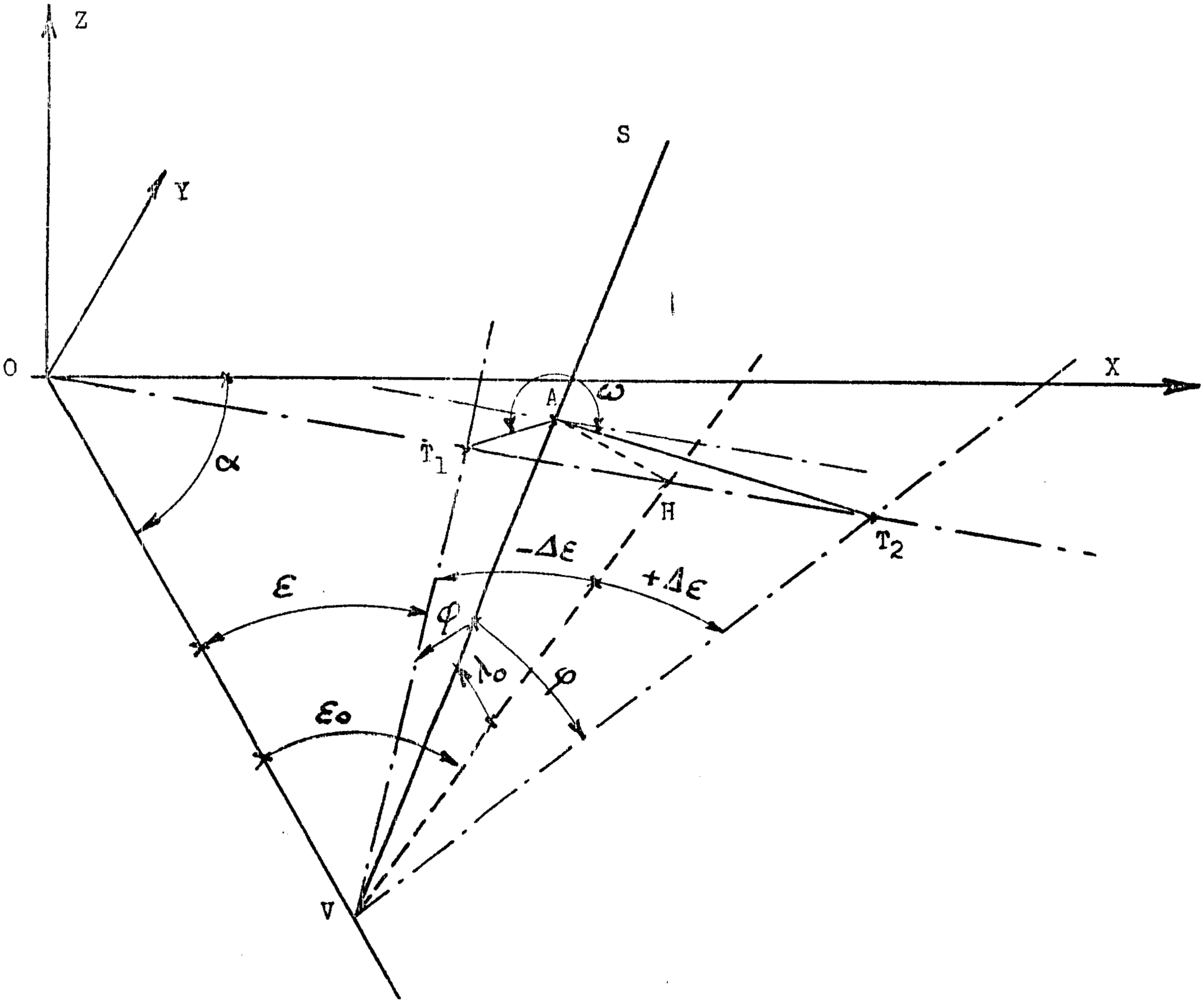
$\Delta\varepsilon$ elongation of the lines of sight VT_1 and VT_2 reckoned from the projection vh

ε_1 elongation of the lines of sight VT_1 reckoned from the direction vo

OT_1 HT_2 trace, in the plane xoy, of the plane normal in A to the axis of spin vs and containing the sun θ.

ω angular velocity of spin of the vehicle.

- Figure I-1 -



Triangulation of the scanning of the Zodiacal Light hence: $$\tan \Delta \varepsilon = \frac{HT}{VH} = \pm \left[\tan^2 \varphi - \tan^2 \lambda_0\right]^{1/2} \cos \lambda_0 \qquad \text{I-5}$$

Form this relationship, one obtains the elongations ε_1 and ε_2 of the directions of sight VT_1 and VT_2, in the plane of the ecliptic, in terms of the elongation ε_0 of the projection of the axis of spin VS at the elevation λ_0 and of the angular distance φ of the revolving line of sight from the said axis of spin VS, as per

$$\varepsilon_1 = \varepsilon_0 - \Delta \varepsilon \qquad \text{I-6}$$
$$\varepsilon_2 = \varepsilon_0 + \Delta \varepsilon \qquad \text{I-7}$$

Also it is seen that the angular position of the revolving plane containing the directions of sight VT_1 and VT_2 is given by:

$$\cos \omega t = \pm \left[1 - \frac{\tan^2 \lambda_0}{\tan^2 \varphi}\right]^{1/2} \qquad \text{I-8}$$

when the lines of sight are contained within the plane of the ecliptic.

Those particular points define the conditions under which the line of sight becomes contained within the plane of the ecliptic. Those conditions must be satisfied at every cycle to acceed to the peak of the brightness of the zodiacal light. But for a general discussion, angular points are not sufficient and the geometry of the trace of the scan must also be considered. For this purpose it is more suitable to express the position of the line of sight in space in terms of its instantaneous elevation λ and of the angular distance φ' of its projection onto the plane of the ecliptic with respect to the projection of the axis of spin located at the elongation ε_0 from the sun. Since the angular distances λ and φ' are also functions of the angular rotation ωt of the plane containing the line of sight, the trace of the scanning becomes defined in term of the time.

In order to avoid expressions in which square roots enter, the instantaneous elevation λ is expressed by:

$$\sin\lambda = \cos\varphi \sin\lambda_0 + \sin\varphi \cos\lambda_0 \sin\omega t \qquad I-9$$

while the elongation φ', reckoned from the elongation ε_0 of the projection of the axis of spin, is given by:

$$\tan\varphi' = \frac{\sin\varphi \cos\omega t}{\cos\varphi \cos\lambda_0 - \sin\varphi \sin\lambda_0 \sin\omega t} \qquad I-10$$

The instantaneous elongation of the projection onto the ecliptic of the line of sight, with respect to the sun, becomes

$$\varepsilon' = \varepsilon_0 + \varphi' \qquad I-11$$

With those expressions one can discuss all the possible modes of scanning and particularly the three main cases.

FIRST CASE

Let's consider, at first, the case where the axis of spin is set vertical to the plane of the ecliptic, that is the case where $\lambda_0 = \pi/2$.

Then the values of λ and φ' are given at any instant by:

$$\sin\lambda = \cos\varphi \qquad I-12$$

$$\tan\varphi' = -\frac{1}{\tan\omega t} \qquad I-13$$

Here the ~~light~~ line of sight scans at the constant elevation:

$$\lambda = \pi/2 - \varphi \qquad I-14$$

from the ecliptic and at constant angular velocity ω.

The elongation of the projection of the line of sight takes all angular values from 0 to 360°. Therefore the scanning of the zodiacal light becomes performed along circles parallel to its plane of symmetry. When the trace of the scanning is selected to coincide with the peak of brightness of the zodiacal light, that is for $\varphi = \pi/2$, then the line of sight passes cyclically through or near the sun with a range of variation of irradiant energy extending from 1 down to 10^{-13}.

FIGURE I-2

SCANNING OF THE ZODIACAL LIGHT

Three principal cases, defined by the position of the axis of spin of the vehicle, bound the scanning of the zodiacal light.

Case I - Axis of spin normal to the ecliptic: elevation : $\lambda_o = 90^\circ$
The scanning follows parallels to the ecliptic at the angular velocity ω of the spin. The obstruction by the earth limits the scanning to an angular range of 180° centered onto the angular distance of the vehicle with respect to the radius ov issued from the sun O and passing through the earth.

Case II - Axis of spin VS directed through the sun O: elongation $\varepsilon_o = 0^\circ$, elevation $\lambda_o = 0^\circ$

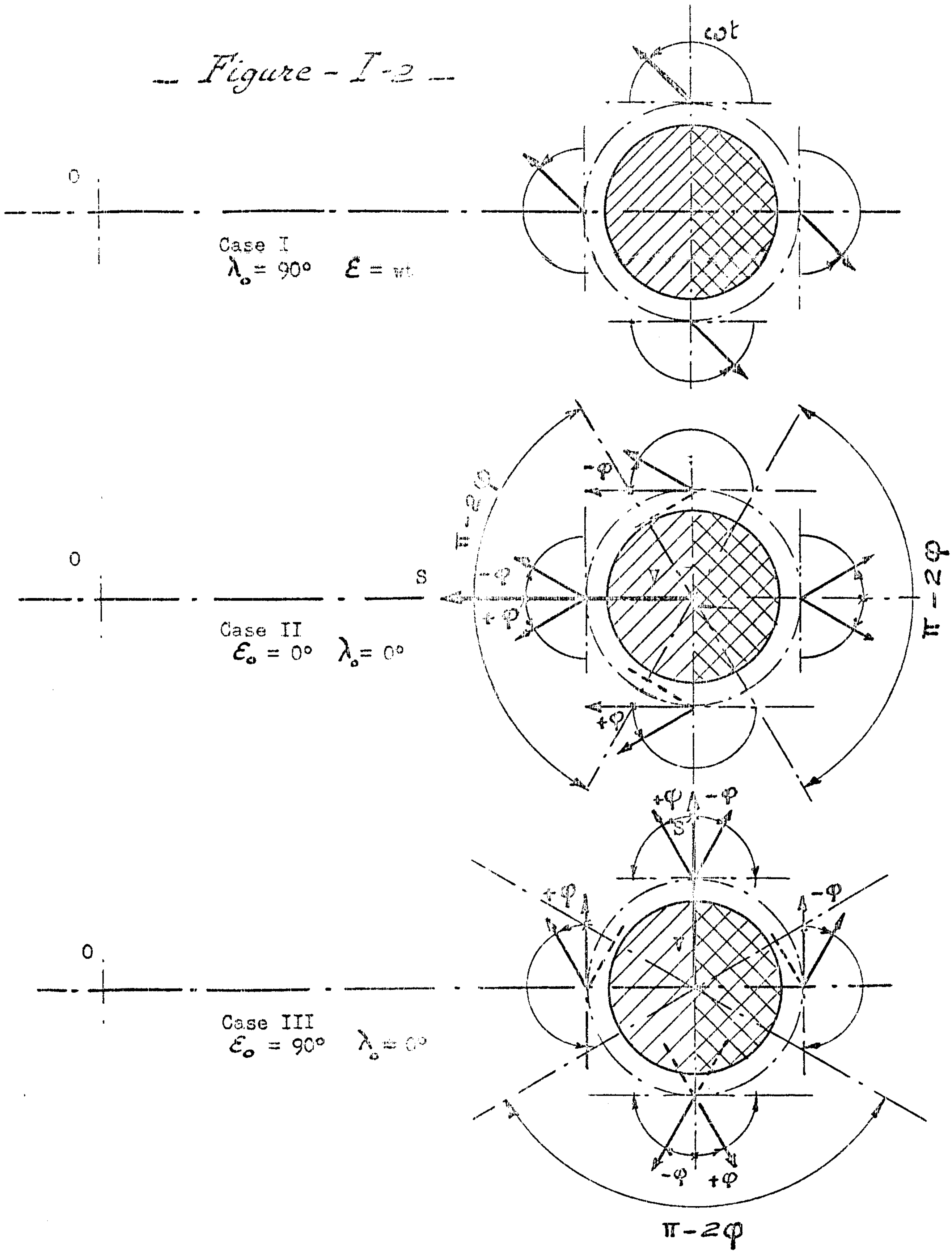
Circular traces of scan, centered onto the sun O, intersecting the plane of the ecliptic at right angles and described at the angular velocity ω. Elongations, in the plane of the ecliptic, equal to + or - the angle φ of the line of sight with respect to the spin axis vs. $\pi - 2\varphi$, arcual length of the trajectory of the vehicle within which two sightings occur, in the plane of the ecliptic, per revolution.

Case III - Axis of spin VS within the plane of the ecliptic and normal to the radius ov issued from the sun O and passing through the earth: elongation $\varepsilon_o = 90^\circ$, elevation $\lambda_o = 0^\circ$

The circular traces of scan intersect orthogonally the plane of the ecliptic at the elongations $\varepsilon_o + \varphi$ and $\varepsilon_o - \varphi$. φ measures the angular distance of the line of sight with respect to the spin axis VS.

$\pi - 2\varphi$ defines the angular distance, along the trajectory of the vehicle, for the occurance of two sightings within the ecliptic.

— Figure - I-2 —



Scanning of the Zodiacal Light

The instrument would have to operate under angular programming with reference to the sun in order to limit the scanning during each revolution to known angular increments $\Delta\omega t$ at programmed elongations ωt. At each revolution, in the vicinity of the sun, the fail safe circuitry would have to turn off the light transducer. Also for about half of each revolution, the line of sight will intersect the earch surface.

During that part of the orbit of the vehicle around the earth where located between the sun and the earth, the scanning will encompass elongations of $+90°$ to $-90°$ for a total angular scan of 180° approximatively centered onto the sun. On the other hand, when the vehicle is located behind the earth, then the elongations of scan are centered at 180° with excursions of 90° on each side. For a short part of the orbit of the vehicle the scanning will encompass a range of elongations varying from 0 to 180°.

This type of scanning is feasible though it entails great emphasis on the fail safe circuit as well as on a rather rigid programming. The data as obtained will have to be processed to obtain the brightness of the zodiacal light in terms of angular distances from its plane of symmetry at given elongations.

SECOND CASE

In opposition to the first case, let's now consider the case where the axis of spin of the vehicle is contained within the plane of the ecliptic $\lambda_0 = 0$ while its elongation maintained at $\varepsilon_0=0$.

Then the relationship

$$\sin\lambda = \sin\varphi \sin\omega t \qquad \text{I-15}$$

and

$$\tan\varphi' = \tan\varphi \cos\omega t \qquad \text{I-16}$$

prevail.

The traces of the scanning become circles centered about the sun, since the axis of spin of the vehicle passes through it. Those circular scans intersect the plane of the ecliptic, thus the plane of symmetry of the zodiacal light, at right angles. For each revolution, the elongations in the plane of the ecliptic are given by:

$$\varepsilon = \pm \varphi$$

and are symmetrical with respect to the sun.

Two consecutive peaks occur at equal interval of time. Being symmetrically located with respect to the sun, those peaks can be compared for their magnitudes. The fail safe circuit will have to act only on accidental setting.

The scan for elongations varying from few degrees to 180 degrees will have to be obtained during two different portions of the orbit of the vehicle. From 0 to 90° elongations, the instrumentation must operate when the vehicle is located between the sun and the earth: the angle φ being set at values varying from 0 to 90° from the axis of spin. On the contrary, for elongations varying from 90 to 180 degrees the instrumentation must operate when the vehicle passes behind the earth. In that case, the angle φ is set at angular distances varying from 90 to 180° with respect to the axis of spin. This mode of scanning appears rather simple and leads to simple data acquisition and presentation.

However, one must note that during passage behind the sun, it will be impossible to reference the brightness of the zodiacal light against the sun for half the period of the orbit of the vehicle. A low drift reference circuit will have to be provided.

THIRD CASE

This case differs from the second one by the elongation $\varepsilon_0 = \pi/2$ of the axis of spin still maintained at the elevation $\lambda_0 = 0$. The instantaneous elevation of the line of sight is given again by:

$$\sin\lambda = \sin\varphi \sin\omega t$$

also its instantaneous relative elongation

$$\tan\varphi' = \tan \cos\omega t$$

But the line of sight becomes contained within the plane of the ecliptic for the two elongations.

$$\varepsilon_1 = \pi/2 - \varphi \qquad \text{I-17}$$

$$\varepsilon_2 = \pi/2 + \varphi \qquad \text{I-18}$$

The trace of the scan is still a circle centered onto the axis of spin and it intersects the ecliptic and therefore the plane of symmetry of the zodiacal light at right angles.

Two consecutive peaks of brightness, at elongations ε_1 and ε_2 respectively, occur at exactly half the period of revolution of the vehicle. They will deffer in intensity on account of the difference in the values of ε_1 and ε_2. However they will be observable one after the other during a single revolution of the vehicle within the orbital angle of:

$$\Delta s \cong \pi - 2\varphi \qquad \text{I-19}$$

centered onto the spin direction.

For the other part of the orbit of the vehicle either one of the elongations ε_1 or ε_2 will be accessible depending whether the vehicle is between the sun and the earth or behind the earth.

Also no sighting will be available within that portion of the orbit corresponding to

$$\Delta s' \cong -(\pi - 2\varphi) \qquad \text{I-20}$$

centered onto the spin axes.

For all eventuality one can conclude that the scanning of the zodiacal light from an orbiting spinning vehicle is feasible. From the three main cases as discussed the second one, corresponding to the axis of spin passing through the sun, seems the simplest to carry out. The third case requires the referencing against two different energy levels corresponding to ε_1 and ε_2. The first case is possible but imposes serious restrictions on the programming and the data must be processed for the exact determination of the peaks.

VALUES FOR BRIGHTNESS AND POLARIZATION OF THE ZODIACAL LIGHT

The brightness of the zodiacal light, its distribution in terms of the elongation from the sun and degree of polarization have been reported by several investigators. The data obtained by Blackwell [Chacaltaya 1958] and lately by Weinberg [Haleakala 1962] are considered as main references.

Using different methods, photographic plates for Blackwell's experiment and direct photoelectric photometric measurements in Weinberg's survey the reported data agree fairly well in spite of the fact that the band passes were different. Blackwell used a OY_2 filter of some 100 millimicrons band pass while Weinberg elected an equivalent 70 A^o band pass interference filter whose peak has been judiciously selected for $\lambda = 5300 A^o$.

The brightness reported by Blackwell [1955], Blackwell and Ingham [1961] and Weinberg [Haleakala 1962] are indicated in terms of the elongations ε in the added graph. The brightness B are referred against the brightness $\bar{B}_{\odot}$ of the integrated solar disk.

Consecutively to the examination of the distribution of the values of the relative brightness as reported, it became evident that, for elongations ε ranging from 2^o to 100^o, the relative brightness $[B/\bar{B}_{\odot}]$ could be tentatively represented by a straight line as shown in the graph. From this graph we have derived a close fitting expression satisfactory for general discussion.

Brightness v.s. elongation follow closely the relationship

$$\log(B/\bar{B}_{\odot}) = \bar{9}.38 - 2.275 \log \varepsilon^o \qquad \text{II-1}$$

FIGURE II-1

BRIGHTNESS $[B/\bar{B}_0]$ OF THE ZODIACAL LIGHT v.s. ELONGATION ε
IN THE PLANE OF THE ECLIPTIC

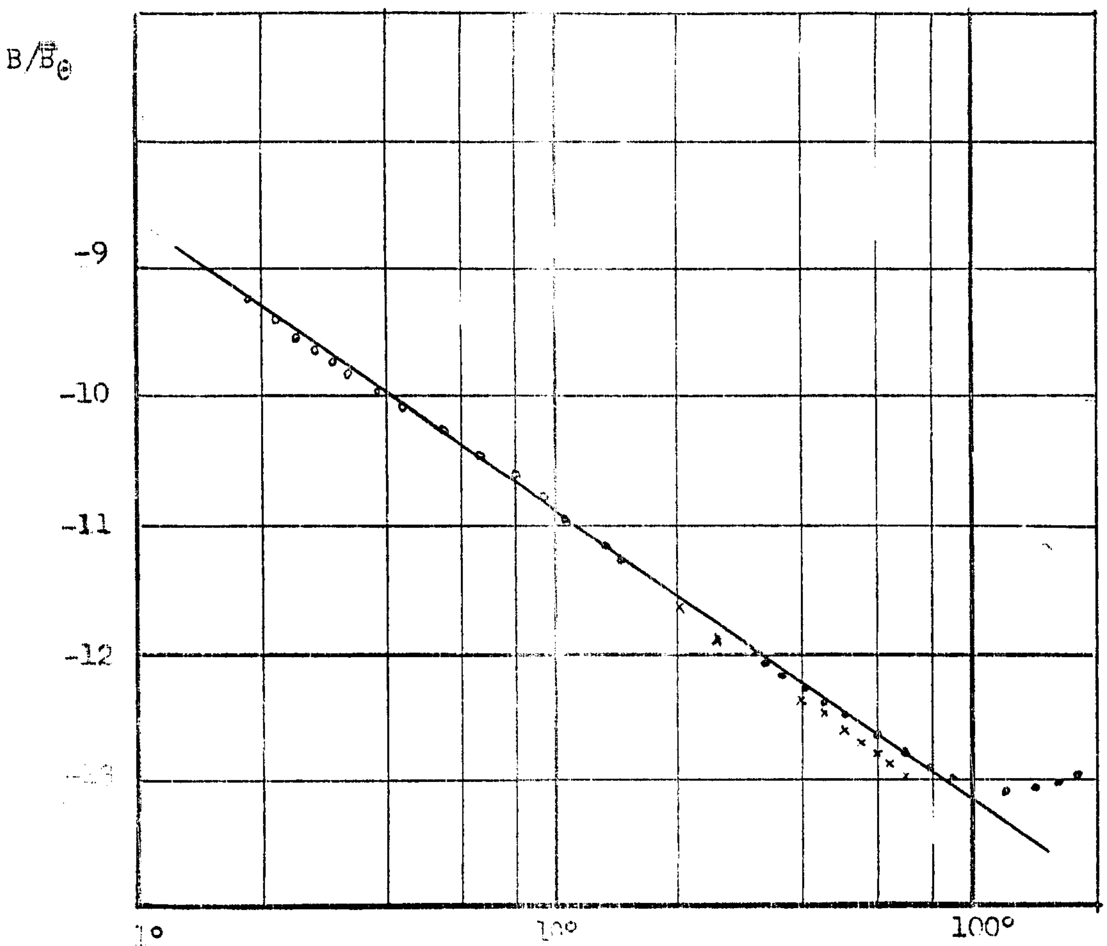
o Blackwell [1955], $\lambda = 6200$ A°, from $\varepsilon = 1.5°$ to $\varepsilon = 15°$

x Blackwell and Ingham [1961a], from $\varepsilon = 20°$ to $\varepsilon = 70°$

o Weinberg, Haleakala [1962] $\lambda = 5300$ A°, from $\varepsilon = 30°$ to $\varepsilon = 100°$

— *Figure II-1* —

Brightness of the zodiacal light vs. elongation



$$\log (B/\overline{B}_0) = \overline{9}.38 - 2.275 \log \varepsilon^\circ$$

which could be used in a method of monitoring the gain of the amplifiers.

Values for the brightness of the zodiacal light extend from $(B/\bar{B}_\odot) = 10^{-9}$ at some 1.4° elongation down to $(B/\bar{B}_\odot) = 10^{-13}$ at around 100° elongation. A four order of magnitude in range.

Though the reported values for the brightnesses are in close agreement one notes a difference in the values of the elongations corresponding to the maximum of the degree of polarization.

Blackwell and Ingham [1961]	elongation	70°	%Pol.	33.2%
Weinberg [1963]	"	70°	"	22.9%

As basis of reference for the energies available outside the earth's atmosphere per unit area and per interval $\Delta\lambda$ of unit wave length one can refer to Johnson's [1954] Solar Spectral Irradiance. Here are finds the following data

λ A°	$erg/cm^2.sec.A^\circ$	$Quanta/cm^2.sec.A^\circ$
5000	198	4.97×10^{13}
5300	195	5.20×10^{13}
5500	195	5.40×10^{13}
6000	181	5.47×10^{13}

Those energies can be compared to the total irradiance per unit area, per minute as reported first by S.P. Langley [1893] at 2.54 cal/cm^2 minute, then revised by C. G. Abbot at 2.1 cal/cm^2 minute and actually accepted as 1.94 cal/cm^2 minute. Those values are of importance since they have been obtained by bolometric measurements.

Related to those data are:

Sun Temperature $T \simeq 5960\,^\circ K$

Sun Radius $R_\odot \simeq 6.95 \times 10^{10}$ cms

Earth to Sun Distance $1\,A.U. \simeq 1.493 \times 10^{13}$ cms which permit an appreciation of the degree of reliability in view of the Planck's formula :

$$E_\lambda = c_1 \lambda^{-5} \left[\epsilon^{\frac{c_2}{\lambda T}} - 1 \right]^{-1} \Delta\lambda$$

with:

$$c_1 = 2\pi h c^2 = 3.74 \times 10^{-5} \text{ ergs/cm}^2 \cdot \text{cm.}$$

$$c_2 = \frac{hc}{k} = 1.4384 \text{ cm} \times \text{degree}$$

III

PARTICULAR CASE FOR DATA ANALYSIS

From the acquired data, relative to the brightness of the zodiacal light, it could be inferred that the distribution of the brilliance follows approximatively a Gaussian distribution centered about its main plane. The symmetry of the zodiacal light about the invariable plane has been noted for many years. Indeed, isophotes have been published by several observers - particularly Roach and his associates, and Divari and Asaad but none of these observers have indicated that the brightness distribution perpendicular to the ecliptic plane can be represented reasonably well by a Gaussian distribution. A Gaussian distribution aspect would lead to an approach for systematical processing and analysis of data which, though a priori redondant, would permit the lowering of the background noise level.

For a given circle of scan centered about the sun or for a trace of scan orthogonal to the main plane of distribution of the brilliance the brightness could be expressed by

$$B_z = B_{zo}\, e^{-K\omega^2} + B_s \qquad \text{III-1}$$

where

B_{zo} = peak brightness of the zodiacal light

B_s = average brightness of the background

at a given elongation ε from the sun.

Taking the first derivative of the equation III-1 yields:

$$B' = -B_{zo} \times 2K\omega\, e^{-K\omega^2} \qquad \text{III-2}$$

an expression in which the average brightness of the background disappears.

The value of the rate of variation B' exhibits two peaks, equal in absolute value but opposit in sign, located at equal distance from the peak of brightness as per:

$$\omega = \pm\sqrt{\frac{1}{2K}} \qquad \text{III-3}$$

FIGURE III-1

APPARENT GAUSSIAN DISTRIBUTION OF THE ZODIACAL LIGHT

$$B_z = B_{z_0} \, e^{-K\omega^2}$$

$K = 1.182$

$\omega \cong$ in unit of $20°$

× Brightness from Weinberg - Haleakala, 4/5 May 1962-

$\lambda = 5300$ A°, $z = 80°$

—— Gaussian Distribution of the Zodiacal Light brightness versus ω

---- B^{I} first derivative of B v.s. ω

—·—· B^{II} second derivative of B v.s. ω

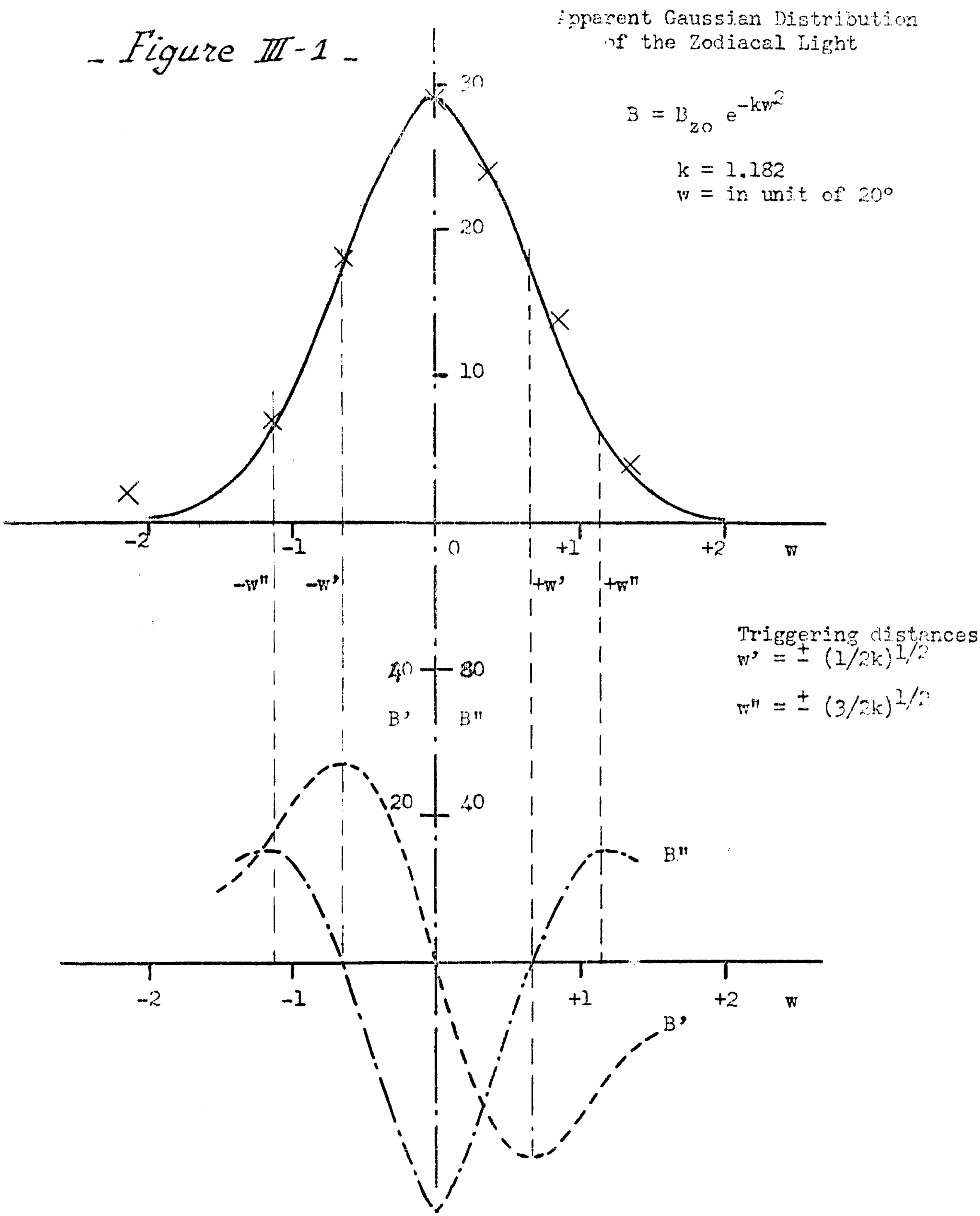

Apparent Gaussian Distribution
of the Zodiacal Light
$B = B_{zo}\ e^{-kw^2}$
k = 1.182
w = in unit of 20°
30
20
10
-2
-1
0
+1
+2
w
-w"
-w'
+w'
+w"
Triggering distances
$w' = \pm (1/2k)^{1/2}$
$w'' = \pm (3/2k)^{1/2}$
40
80
B'
B"
20
40
B"
-2
-1
+1
+2
w
B'

Figure III-1

yielding

$$B_m' = \pm B_{Zo} \sqrt{2K} \; e^{-1/2} \qquad \text{III-4}$$

or

$$B_m' = \pm B_{Zo} \times 0.8582 \sqrt{K} \qquad \text{III-5}$$

B' also passes through zero when the measured brightness B_Z reaches the maximum. Hence one disposes of three triggering signals to accept and store data for further evaluation. However the signal in B_m' may be rather broad leading to a relative uncertainty in the determination of the location of the sampling.

However taking the second derivative of the brightness yields the expression:

$$B'' = B_{Zo} \times 2K \left(2K\omega^2 - 1\right) e^{-K\omega^2} \qquad \text{III-6}$$

which becomes zero for the value $\omega = \pm\sqrt{\frac{1}{2K}}$

This second derivative passes through two symmetrical maxima at distances $\omega = \pm\sqrt{\frac{3}{2K}}$. Being broad peaks, a triggering signal cannot be derived but a release level can be achieved.

The third derivation of the brightness function

$$B''' = B_{Zo} \times 4K^2 \left[3 - 2K\omega^2\right] \omega \, e^{-K\omega^2} \qquad \text{III-7}$$

becomes zero for the values of ω :

$$\omega = 0$$
$$\omega = \pm\sqrt{\frac{3}{2K}}$$
$$\omega = \pm\infty$$

Hence the zero of the third derivative occuring at the distances $\pm\sqrt{\frac{3}{2K}}$ can command sampling and logging of the function B_Z which has, at those distances, the value

$$\left(B_Z\right)_{\left(\pm\sqrt{\frac{3}{2K}}\right)} = B_{Zo} \, e^{-3/2} + B_s \qquad \text{III-8}$$

To racipulate, the following samplings can be obtained in function of the distance ω :

$$\omega = 0 \qquad B_{z_0} + B_s \qquad \text{III-9}$$

$$\omega = \pm\sqrt{\frac{1}{2K}} \qquad B_{z_0} e^{-1/2} + B_s \qquad \text{III-10}$$

$$\omega = \pm\sqrt{\frac{3}{2K}} \qquad B_{z_0} e^{-3/2} + B_s \qquad \text{III-11}$$

Therefore it is possible to process the following data

$$2(B_{z_0} + B_s) - 2(B_{z_0} e^{-1/2} + B_s) = 2 B_{z_0}(1 - e^{-1/2}) \qquad \text{III-12}$$

$$2(B_{z_0} + B_s) - 2(B_{z_0} e^{-3/2} + B_s) = 2 B_{z_0}(1 - e^{-3/2}) \qquad \text{III-13}$$

$$2(B_{z_0} e^{-1/2} + B_s) - 2(B_{z_0} e^{-3/2} + B_s) = 2 B_{z_0}(e^{-1/2} - e^{-3/2}) \qquad \text{III-14}$$

to achieve a reduction in the noise.

The proper logistic of the computation as outlined should be studied in more details and in regards with the electronics compatibility. Also the degree of compatibility of the gaussian distribution for the brightness of the zodiacal light is not readily available from the actual data. If the agreement was found valid within the degree of incertitude of the experimental data one should expect an enhancement of the overall acuracy while obtaining a rather simple analytical presentation for further study of the causes and mechanisms involved.

This possibility arises from the examination of the Weinberg's scan recording for total brightness along an almucantar taken at the zenith distance z=80° [Halea~~t~~kala] - May 4/5 - 1962 for λ =5300 A°.

From this recording we have found the following gaussian parameters:

$K \cong 1.172$

$\omega \cong$ unit of angle of scan equal to ~ 20°

$B_{z_0} \cong 29$ arbitrary units, maximum amplitude

The gaussian distribution has been plotted for that case as well as its first and record derivatives D' and D''.

ATTENUATION OF THE NOISE CAUSED BY STARS CROSSING THE FIELD OF VIEW

An attenuation of the equivalent noise resulting from the input of radiant energy due to a star crossing the field of view must be incorporated into the circuitry.

With the presence of a field diaphragm and in the absence of vignetism, marginal aberrations, secondary reflections, etc. in the telescope collecting the radiated energy, the crossing of a star would produce a square wave pulse. In fact, the square wave pulse appears as more or less rounded. The pulse rise is limited and admits a finite time constant. In any event, the equivalent time constant t_{cs} for a crossing star cannot exceed one fourth of the incremental time corresponding to a diametral crossing of the field of view U.

This maximum time constant t_{cs} becomes readily defined in terms of both angular velocity or frequency ν of the spin of the vehicle and field of view U :

$$t_{cs} \leq \frac{1}{2\pi\nu}\left(\frac{U}{4}\right) \qquad \text{IV-1}$$

In the present case where the frequency ν is imposed at

$$\nu = 1/2 \text{ cycle/sec.}$$

one obtains

$$t_{cs} \leq 1.41 \times U \text{ millisecond} \qquad \text{IV-2}$$

This time constant can be readily compared against that associated to the maximum rate of rise deduced from the equivalent gaussian distribution of the zodiacal light along a direction normal to its principal axis. From the Weinberg's data - Haleakala 1962 - it is possible to assign a rate of rise of:

$$\left(\frac{dB_{Z0}}{B_{Z0}}\right) = \sqrt{2K} \times e^{-1/2} \times d\omega \qquad \text{IV-3}$$

for a unit of angular distance $\omega \cong 20$ degrees and a constant $K \cong$ 1.182. Thus the time constant to be assigned to the scan of the zodiacal light appears to be of the order of

$$t_{cz} \cong 118.5 \text{ milliseconds}$$

for an angular velocity of spin given at 1/2 cycle per second.

The ratio of the two time constants

$$t_{cs}/t_{cz} \leq 0.0119 \times U$$

suggests that the angular field of view be maintained as small as compatible for the necessary signal to noise ratio. Also in view of the approximate difference of two orders of magnitude, it is appropriate to incorporate a feed back in the amplifier to attenuate signals having time constants smaller than 1.41 x U milliseconds.

The amplifier should be designed as a low pass band system rejecting those frequencies above 1/1.41 x U. A refined circuitry could be added to the feed back to impose a decrease of the signal output by an amount proportioned to the amplitude of the pulse.

However the application of this method introduces a shift and a reduction of the amplitude of the peak. Phase shift and amplitude deviation become parameters of the system and can be calibrated. Those parameters will have to be analyzed and determined exactly with respect to the final accuracy.

V

ANGULAR PRECISION v.s. ANGLE OF FIELD OF VIEW

To track exactly the distribution of the zodicacal light brightness and the position of its peak, the angular field of view U of the instrument should be made as small as possible. Yet, this condition cannot be achieved since there is a minimum requirement of radiated energy to secure proper performance of the light transducer. Opening the angular field of view, to collect more energy, cannot be achieved without loosing some definition in the energy distribution and imposing a more or less severe truncature of the contour around the peak.

On the contrary, the necessity to reduce the relative percentage of the light energy from a star crossing the field presents an argument in favor of a large field of view.

A decision for the value of the field of view of the instrument cannot be taken at first glance. At least some preliminary analysis of its effects should be done.

If one considers the noted apparent gaussian distribution of the brightness of the zodiacal light tentatively expressed by

$$B_z = B_{zo}\, e^{-K_o \omega_o^2} \qquad V-1$$

wherein $K_o \cong 1.18$

$\omega_o \cong$ unit of 20 degrees

it is possible to evaluate the attainable uncertainty in angular position versus the resolving power of the instrumentation and this under ideal conditions.

For the sake of clarity, the angular unit ω_o , as defined and equivalent to some 20 degrees, must be converted into conventional unit of angular distances, ω degrees, by changing the constant K_o one obtains the new constant K as per

$$K = [1.18]\,[20]^{-2} = 2.95 \times 10^{-3} \qquad \text{V-2}$$

since K_o has been evaluated at $K_o = 1.18$

Through differentiation of [V-1] one obtains the relative uncertainty

$$\delta B = \frac{dB}{B} = -2K\omega\, d\omega \qquad \text{V-3}$$

This uncertainty δB cannot be smaller than the relative resolving power of the instrument which, for the case of an analog system, reaches a limit of δr =0.001

Hence the uncertainty in angular position:

$$\delta\omega = -(2K\omega)^{-1}\,\delta B \qquad \text{V-4}$$

This expression indicates readily that around the peak, that is when ω tends twoard zero, the angular definition decreases very readily. For instance, taking a value $\omega = 1^{\circ}$ the uncertainty in $\delta\omega$ takes the value:

$$\delta\omega = \pm\, 0.001\left(2 \times 2.95 \times 10^{-3}\right)^{-1} = \pm\, 0.17 \text{ degree}$$

as the limit

On the other hand, for an instrumental precision of 0.01 or 1%, the angular position of the peak becomes determined within a limit of

$$\left|\delta\omega\right|_{.01} = \pm\, 1.7 \text{ degree}$$

This relatively low resolution in angular position had to be expected around the plateau of the peak as it is inherent in this type of distribution.

However, noting that the maximum tangent for the gaussian distribution occurs at

$$\omega' = \pm (2K)^{-1/2}$$

suggests that the location of the peak be implemented by the data available at this value.

At that point the limit of angular uncertainty becomes

$$|\delta\omega'| = \pm (2K)^{-1/2} \delta B = \pm 13.04 \times 10^{-3} = \pm 0.013 \text{ degree}$$

yielding, for an instrumental precision of 1%, an angular definition of some $\pm$ 0.13 degrees.

These fundamental aspects should be considered specifically in the study and design of the electronics associated with data acquisition and processing. Indeed there is a very favorable argument for process sampling at the angular distance $\omega' = \pm(2K)^{-1/2}$ since for that position the second derivative passes through zero and therefore can trigger with precision the sampling at that position.

Those ideal conditions cannot be achieved since the angular field of view U must be assigned a value compatible with the requirement for adequate signal strength while decreasing the relative amplitude of the pulse caused by a crossing star.

An analysis is necessary to evaluate the blunting of the peak of brightness of the zodiacal light and the loss of precision in angular positioning consecutive to a given angular field of view.

FIGURE V-1

COLLECTED ENERGY AS A FUNCTION OF THE ANGULAR FIELD OF VIEW U AND OF THE ANGULAR DISTANCE OF THE ZODIACAL LIGHT

OX, OY orthogonal axes of coordinates centered onto the axis of the field of view U defined by the circular operture of the field diaphragm of diameter $2\omega_u = U$

B_{Zo} axis of maximum brightness of the zodiacal light

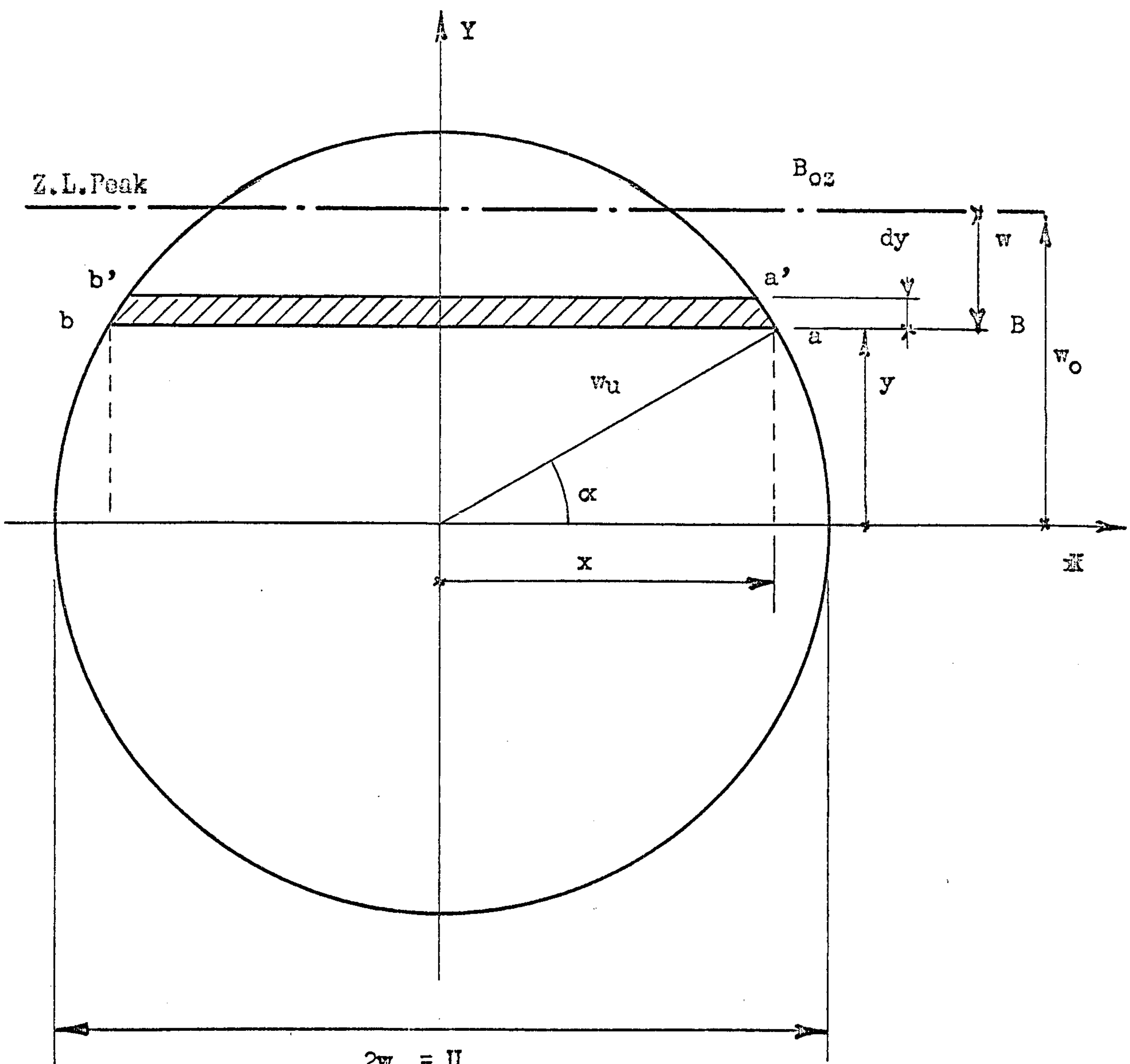
W_o distance of the axis B_{oz} of maximum brightness from the axis of reference ox

Y distance, from the axis of reference ox, of the zone of brightness B at the distance W from the peak B_{oz}

$aa'bb'$ increment of field area determined by its half length x and the increment dy in the distance y

α the angle made by the radius $Oa = W_u$ and the axis ox.

— *Figure V-1* —



Collected energy as a function of the angular field of view U and of the angular distance of the peak of the zodiacal light

For this purpose one can make use of the apparent gaussian distribution of the zodiacal light which, around the peak, can be expressed by the equivalent series

$$B = B_{zo}\, e^{-K\omega^2} \cong B_{zo}\left[1 - K\omega^2 + \frac{(K\omega^2)^2}{2!} - \dots\right. \qquad \text{V-5}$$

Near the peak one can neglect the terms of higher order than the second. This assumption is permissable when the angle of field of view U remains limited to a few degrees.

Considering the practical case for which the field of view being limited by a circular field diaphragm one can express the elementary flux of energy dE corresponding to an infinitesimal increment of area of the field as per:

$$dE = B \times 2\, x\, dy \qquad \text{V-6}$$

The brightness B of the zodiacal light can be defined in terms of the brightness B_{oz} of the peak and of its distance ω from the elementary surface area aa^1bb^1. A transformation of coordinates yields:

$$\omega = y - \omega_o \qquad \text{V-7}$$

hence

$$B \cong B_{zo}\left[1 - K(y - \omega_o)^2\right] \qquad \text{V-8}$$

and

$$dE \cong 2 B_{zo}\left[1 - K(y - \omega_o)^2\right] x\, dy \qquad \text{V-9}$$

Expressing the coordinates x and y in terms of the angular field of view U limited by the diaphragm,

that is:

$$x = \frac{1}{2} U \cos\alpha \qquad \text{V-10}$$

$$y = \frac{1}{2} U \sin\alpha \qquad \text{V-11}$$

leads to the final value of the incremental energy

$$dE \cong 2 B_{zo}\left[1 - K\left(\frac{U}{2}\sin\alpha - \omega_o\right)^2\right] \frac{U^2}{4} \cos^2\alpha \times d\alpha \qquad \text{V-12}$$

An integration within the limit $-\pi/2$ to $+\pi/2$ gives the total flux of energy passing through the field diaphragm:

$$E \cong B_{zo} \frac{U^2}{2} \int_{-\pi/2}^{+\pi/2} \left[1 - K\left(\frac{U}{2}\sin\alpha - \omega_o\right)^2\right] \cos^2\alpha \times d\alpha \qquad \text{V-13}$$

Performing the integration, one obtains:

$$E \cong B_{zo} \left(\frac{\pi U^2}{4}\right)\left[\left(1 - K\omega_o^2\right) - K\left(\frac{U}{4}\right)^2\right] \qquad \text{V-14}$$

for the expression of the total flux passing through the field diaphragm.

It is seen that the accepted energy is made of two quantities: one variable and the other constant. The quantity

$$E_{\omega_o} = B_{zo} \left(\frac{\pi U^2}{4}\right)\left(1 - K\omega_o^2\right) \qquad \text{V-15}$$

is truly representative of the brightness B existing at the distance ω_o from the location of the peak and proportional to the square of the angle of field of view.

The constant quantity, independent of the distance ω_o of peak, appears to be also proportional to the solid angle of the instrumental acceptance and has the relative ~~magnitude~~ magnitude.

$$\frac{\Delta E}{E_{\omega_o}} = -\frac{1}{16} \times \frac{KU^2}{1 - K\omega_o^2} \qquad \text{V-16}$$

The evaluation of the relative loss for the peak intensity, occuring for $\omega_o = 0$, resulting from an angular field of view of 4 degrees leads to:

$$\left(\frac{\Delta E}{E_{\omega_o}}\right)_{4^\circ} = -2.95 \times 10^{-3} = -0.295\ \%$$

and down to

$$\left(\frac{\Delta E}{E_{\omega_o}}\right)_{3^\circ} = -0.166\ \% \quad \text{for } U = 3 \text{ degrees}$$

The relative deviation imposed by the value of the field of view equals the limit of resolving power, 0.001 for analog instrument, at :

$$U = 2.^{\circ}33$$

On the other hand the angular relative sensitivity is again given by

$$\delta E \cong -2K\delta\omega_0$$

and appears as unaffected by the magnitude of field of view.

Therefore it can be stated that the value of the field of view does not distort the distribution of the brightness it introduces solely a small reduction of the peak magnitude. The percentage of reduction can be treated as a parameter of over all transmission through the optics.

Yet due consideration must be given to the fact that the absolute variation ΔE , for a given change $\Delta\omega_0$ being proportional to the solid angle of view, one can by increasing U minimize readily the relative influence of the equivalent noise imput of the transducer.

From this development one arrives at the conclusion that the practical angle of field of view will be mostly limited by the characteristics of the optics and possible angular acceptance.

VI

SURFACE BRIGHTNESS AND ENERGY FLUX

An important objective of the present investigation is the measurement of the surface brightness of the zodiacal light in absolute C.G.S. units, more precisely its specific intensity in terms of ergs. cm^{-2} sec^{-1} $steradian^{-1}$, $(\Delta\lambda)^{-1}$ We do this by comparing the flux from the sun at the earth distance with the energy received in an acceptance cone $\Delta\omega$ from a point (ε,β) in the zodiacal light. The flux from each cm^2 of the solar surface is given by

$$\pi \bar{B}_{\odot} = 2\pi \int_{\theta=0}^{\theta=\pi/2} I(0,\theta) \sin\theta \cos\theta \, d\theta \qquad \text{VI-1}$$

Now $I(0,\theta)$, the specific intensity of the radiated energy at the surface of the sun, depends on the angle θ and the wavelength λ.

The limb darkening law, for monochromatic radiation, can often be expressed as a function of the form:

$$\frac{I_\lambda(0,\theta)}{I_\lambda(0,0)} = A_\lambda + B_\lambda \cos\theta + C \cos^2\theta \qquad \text{VI-2}$$

while for total radiation we have a formula of the form:

$$\frac{I(0,\theta)}{I(0,0)} \cong a + b \cos\theta \qquad \text{VI-3}$$

with $a \sim 0.4$ and $b \sim 0.6$

The flux, E, at the earth distance is given by

$$E = \pi \bar{B}_{\odot} \left(\frac{R_{\odot}}{L}\right)^2 \qquad \text{VI-4}$$

where

R_0 = radius of the sun

L = distance of the earth from the sun

If $\Omega_{\odot}$ is the solid angle filled by the sun as seen from the earth, and $\Delta\omega_z$ is the solid angle measured in the zodiacal light, then the quantities we compare observationally are $\bar{B}_{\odot}\Omega_{\odot}$ and $B_{ZL}(\varepsilon,\beta)\Delta\omega_z$. Then we may determine the ratio:

$$B_{ZL}(\varepsilon,\beta)/\bar{B}_{\odot} \qquad \text{VI-5}$$

and thus express $B_{ZL}(\varepsilon,\beta)$ on an absolute scale, or in terms of some other unit such as number of tenth magnitude stars per square degree.

The parameters for the observation of the brightness of the zodiacal light having been defined specifically, it appears appropriate to review the fundamentals entering into designing, testing and control of the instrumentation.

In this study, one is constantly faced with the obligation of converting brightness into available energy per unit area at given distance from the source and also to evaluate th. degree of reliability of recorded data. Gross errors could be easily introduced in the determination of both optical and electrical parameters of the instrumentation. Moreover the behavior of the instrument should be evaluated in terms of spectral distribution of the light energy instead of one or several discrete band passes to establish its range of application.

As a consequence, it has been deemed necessary to review the fundamentals relating the emission of light energy, brightness of the source and flux of energy per unit of time impinging onto the unit area of a distant collecting surface.

The Lambert's law states that the energy B_{α} emitted per unit area of the surface of a perfect diffusor, along a direction making an angle α with the normal to the surface, is equal to the product of the intensity of emission B_O in the direction of the normal by the cosine of the angle α . Thus the relationship:

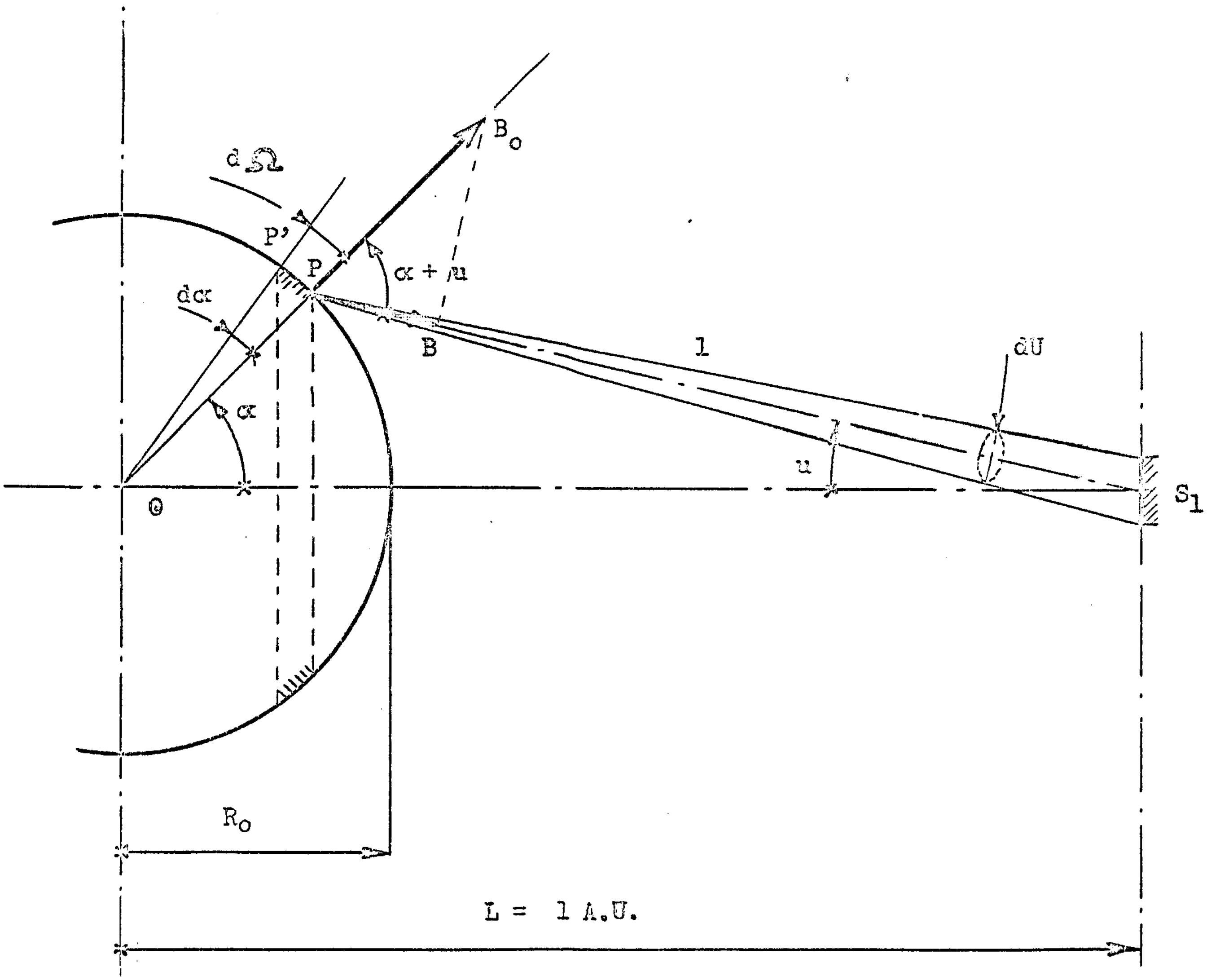
$$B = B_0 \cos\alpha \qquad \text{VI-6}$$

FIGURE VI-1

ENERGY RADIATED BY THE SUN ONTO A UNIT SURFACE
AT THE DISTANCE OF THE EARTH

O center of the sun - S_1 unit area at the distance L of the earth from the sun and normal to the direction OS_1 - dU solid angle under which the area S_1 is seen from a point P_1 of the sun surface - u the angle made by the direction PS_1 with respect to the axis of reference OS_1 - R_o the radius of the sun - α the angle made by the radius OP with the axis OS_1 - $d\alpha$ the angular increment subtending the arcual increment PP' - $d\Omega$ the incremental spherical area corresponding to the increment $d\alpha$ of the angle α.

_ Figure VI - 1 _



Energy radiated by the sun onto a unit surface
at the distance of the earth

from which one obtains the analytical expression for the emitted energy within an increment of solid angle of the form $2\pi \sin\alpha \, d\alpha$:

$$dE = B_o \, 2\pi \sin\alpha \cos\alpha \, d\alpha \qquad \text{VI-7}$$

The integration within the limits o to $\pi/2$ defines the total emitted energy E as:

$$E = \pi B_o \qquad \text{VI-8}$$

The total energy E emitted per unit area of a perfect diffuser is equal to π radians times the intensity B_o of the emission of energy along the direction normal to the surface. Conversely the brightness B_o of the emitting surface is defined as equal to the total emitted energy E divided by π radians, or the flux of energy emitted per steradian.

From those fundamentals, one can relate the energy impinging upon a surface to the distance and brightness of the source.

In particular it is possible to establish immediately an analytical relationship to determine the amount of energy radiated by the sun and impinging upon the unit area at the distance of the earth.

For this purpose, one can write the incremental energy dE corresponding to an incremental radiating surface of the sun as per:

$$dE = B_o \cos(\alpha + u) \times \frac{S_1}{\rho^2} \; 2\pi R^2 \sin\alpha \, d\alpha \qquad \text{VI-9}$$

where $2\pi R^2 \sin\alpha \, d\alpha$ defines an increment of the emissive spherical surface of the sun which is centered upon the axis passing through the center of the sun and the remotely located unit area S_1.

The solid angle under which the irradiated surface S_1 is seen from the elementary emissive surface is measured by:

$$dU = \frac{S_1 \cos u}{\rho^2} \qquad \text{VI-10}$$

The density of emission or brightness at the angular distance $\alpha + u$ of the collecting surface S_1 from the normal to the elementary emissive surface of the sun has the value

$$B = B_o \cos(\alpha + u) \qquad \text{VI} - 11$$

For computation, the following trigonometric relationships

$$\ell^2 = L^2 + R^2 - 2RL\cos\alpha \qquad \text{VI} - 12$$

$$\cos u = \frac{L}{\ell}\left(1 - \frac{R}{L}\sin\alpha\right) \qquad \text{VI} - 13$$

$$\sin u = \frac{R}{\ell}\sin\alpha \qquad \text{VI} - 14$$

exist.

Therefore, the increment of energy dE falling onto the surface S_1 becomes

$$dE = 2\pi B_o \frac{R^2}{L^2}\left(1 + \frac{R^2}{L^2}\right)^{-2}\left[1 - \frac{2R}{L}\left(1 + \frac{R^2}{L^2}\right)^{-1}\cos\alpha\right]^{-2} \times \ldots$$
$$\ldots \times \left[\left(1 - \frac{R}{L}\right)^2 \sin\alpha\cos\alpha - \frac{R}{L}\left(1 - \frac{R}{L}\cos\alpha\right)\sin^3\alpha\right] \times d\alpha \qquad \text{VI} - 15$$

Noting that the radius $R_\odot$ of the sun and the distance L of the earth to the sun have the respective values

$$R_\odot \cong 6.95 \times 10^{10} \text{ cm.}$$
$$L \cong 1 \text{ A.U.} = 1.493 \times 10^{13} \text{ cm}$$

the ration R/L stands at

$$R/L \cong 1/2.149 \times 10^2$$

while its square amounts to

$$(R/L)^2 \cong 1/4.62 \times 10^4$$

One can develop in series the expression for the increment of flux dE and neglect the terms of order higher than the second order. This operation yields the expression:

$$dE \cong 2\pi B_o (R/L)^2 (1 - 2R^2/L^2) \Big[\sin\alpha \cos\alpha - \frac{R}{L}\sin^3\alpha - \ldots\ldots$$

$$\ldots - \frac{3R^2}{L^2}\left(1 - \frac{4R^2}{3L^2}\right)\sin^3\alpha\cos\alpha + \frac{2R}{L}\left(1 - \frac{2R^2}{L^2}\right)\cos^2\alpha \sin\alpha + \ldots\ldots$$

$$\ldots + \frac{5R^2}{L^2}\left(1 - \frac{16R^2}{5L^2}\right)\cos^3\alpha \sin\alpha \ldots\ldots \Big] d\alpha \qquad \text{VI-16}$$

Upon integration over the limits 0 to $\pi/2$, one obtains the expression:

$$E \cong \pi B_o \frac{R^2}{L^2}\left(1 - \frac{2R^2}{L^2}\right)\Big[1 + \frac{R^2}{L^2}\left(1 - \frac{6R^2}{L^2}\right) \ldots\ldots \qquad \text{VI-17}$$

for the energy E, radiated by the sun, impinging upon a unit area of $1\ cm^2$ at the distance of the earth from the sun.

In view of the small magnitude of the ratio R/L, one can consider the approximate expression:

$$E \cong \pi B_o (R/L)^2 \qquad \text{VI-18}$$

as valid.

Hence the brightness $B_\odot$ of the sun surface is unequivocally related to the radiant energy E received per unit surface, per second, at the distance of the earth by

$$B_\odot \cong \frac{E}{\pi\left(\frac{L}{R_\odot}\right)^2} = \frac{E}{3.14 \times 4.62 \times 10^4} \qquad \text{VI-19}$$

The designing parameters for the instrument are entirely accessible from consideration of the temperature of the source and reported data can be evaluated for their degree of departure.

Chapter VII

LIGHT TRANSDUCER REQUIREMENTS

The values of the brightness of the zodiacal light recorded by both D. E. Blackwell - Chacaltaya 1958 and lately by I. L. Weinberg - Haleakala 1961 are in close agreement. The agreement prevails in spite of the fact that the equivalent band pass of the filters used are in a 10/1 ratio or thereabout. For the purpose of evaluating the required characteristics of the light transducer and to determine its working conditions one can consider a straight line log-log function between brightness and elongation ε from the sun. This is a rather convenient aspect since it permits to adjust the sensitivity of the transducer, on a long time constant, and/or the amplification gain in term of the log of the elongation.

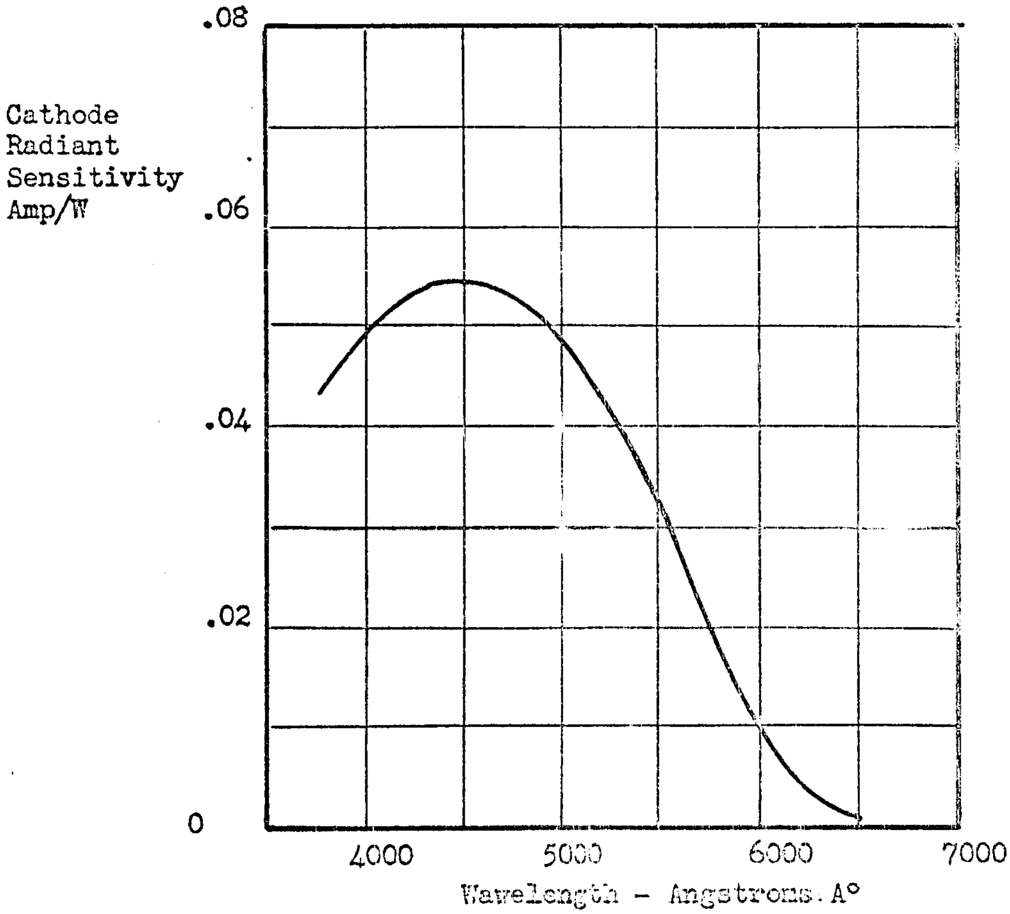
The range of brightness extends from $B/\bar{B}_{\odot} = 1 \times 10^{-9}$ for an elongation of 1.5° down to 1×10^{-13} at an elongation of some 160° approximatively. Hence a $1 \times 10^{4}/1$ ratio of brightness is to be considered as the ultimate goal. For elongations varying from 10° to 180° the range of brightness falls already to a 100/1 ratio or thereabout.

At large elongations, the extremely low level of radiant energy per cm^2 impinging upon the light transducer imposes rather severe restrictions as to the equivalent radiant energy for the noise input of the transducer. The equivalent noise input becomes the predominant factor in the selection of the light transducer in order to avoid considerable difficulties in the electronic circuitry. A minimum of 4/1 ratio between signal and noise levels should be preserved.

A preliminary survey of the available high quantum efficiency light transducers indicates that the VENETIAN BLIND type photomultipliers are

- Figure VIII-1 -

Spectral Response - Typical



Typical Spectral Response Characteristics
of the E.M.R. Photomultiplier Type 541A - 01 - 14
Venetian Blind Type - 14 Dynodes

characterized by a very low equivalent noise input and low dark current. Close to 2 orders of magnitude lower levels are noted.

Among others, the E.M.R. - Type 541A-01-14 seems particularly well suited for this program. The main characteristics for a typical tube are quite impressive.

a) Photocathode characteristics

Quantum efficiency (φ) at 4100 A°	15%
Cathode luminous sensitivity (S_k)	65 A/lm
Cathode peak radiant sensitivity (σ_k)	.055 A/W
Cathode radiant sensitivity at 5500 A°	.033 A/W

b) Multiplier Phototube characteristics

Voltage required for current amplification (G)

G	10^5	10^6	10^7	10^8
V	1600	2100	2720	3450 volts

Dark current (i_D) at 20°C at current amplification G

G	10^5	10^6	10^7	10^8
i_D	2.2×10^{-10}	2.0×10^{-9}	2.0×10^{-8}	2.3×10^{-7}

- Equivalent anode dark current input at 20°C at current amplification of 10^6

  Radiant at 4500 A° (ε_D) 3.7×10^{-14} W

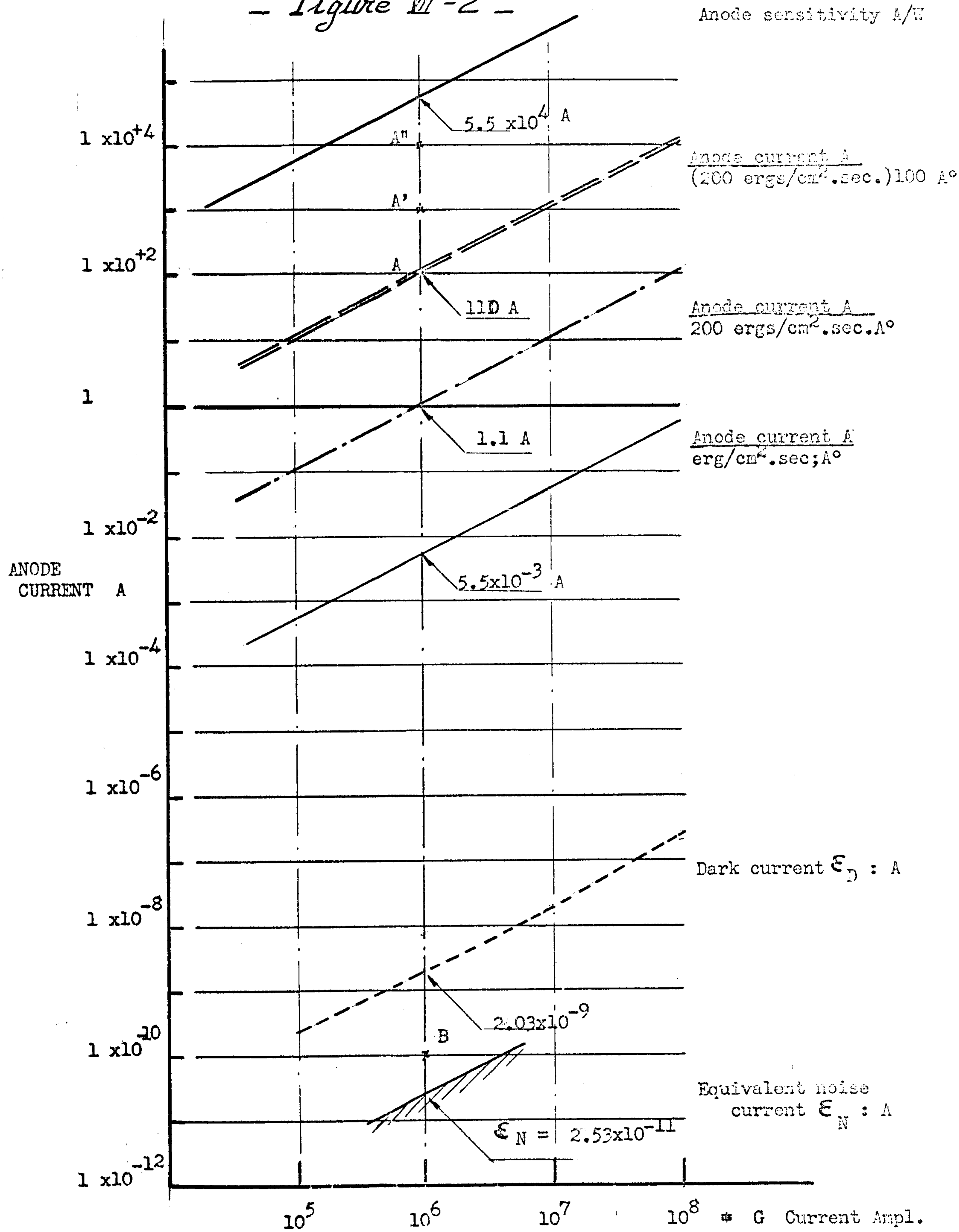
- Equivalent noise input at current amplification of 10^6 at 20°C

  Radiant at 4500 A° (ε_N) 4.6×10^{-16} W

c) Environmental

Shock	100 g, 11 multisecond duration
Vibration	30 g, 20 to 3000 cps
Temperature	-55 to 75°C

Figure VII-2



E.M.R. Photomultiplier Type 541A - 01 - 14 Typical

From those typical data which can be improved through selection of the tube it is possible to establish the tentative performances of the system under practical condition.

The normal performances have been plotted for ease of evaluation in terms of impinging energy and current amplification. Under a cathode current amplification of 10^6, achieved for an anode voltage of some 2100 volts, the equivalent noise current reaches a value of

$$\mathcal{E}_N = 2.53 \times 10^{-11} \quad \text{amper}$$

while the dark current at the anode represents some

$$i_D = 2 \times 10^{-9} \quad \text{amper}$$

On the graph, anode currents are plotted in function of the amplification of the tube and for different levels of impinging radiant energies.

It must be pointed out that the anode current characteristics, nearly straight lines v.s. the amplification, are given for radiant energies expressed in ergs per second falling onto the cathode. Those energies are compared against the radiant energy of the sun falling onto a unit surface of one square centimeter, outside the earth atmosphere, per angstrom and per second.

The reference line, dash and dot, for the anode current passing at 1.1 amper for $G = 10^6$ and corresponding to an incident radiant energy of 200 ergs per second has been traced in due reference to the energy of some 195-198 ergs per cm^2, per second and per angstrom as reported by Johnson 1954 for the solar flux outside the earth atmosphere in the wave length range of 5300 to 5400 angstroms.

Therefore, for a practical passing band of 100 A°, defining an energy of some 2×10^4 ergs per second, one obtains an equivalent anode current of 110 ampers at $G = 10^6$. The double dash line passing through the point A, 110 amps - $G = 10^6$, forms the base line for the design.

Chapter VIII

THE DIFFERENCE TO THE SUM RATIO

and THE MEASUREMENT OF THE DEGREE OF POLARIZATION

The determination of the degree of polarization of the zodiacal light requires that the fraction of the light energy which is plane polarized be extracted from the total energy. Different procedures can be devised. In all cases, the determination of the fraction represented by the polarized energy, imposes the measurement of the energy passing through an analyzer in function of known angular positions of its axis of polarization. At least two observations, at two known angular positions, are required.

The necessary conditions for observation and the degree of accuracy which can be secured can be analyzed readily.

Let's consider an axis of reference OX from which all angular positions are measured. Then, the direction of the amplitude A_0 of the plane polarized light is determined by its angular distance β from the axis OX.

If one places, on the light beam, an analyzer P_1 with its axis of polarization at the angular distance γ_1 from the reckoning direction, the component of the plane polarized amplitude along the direction P_1 is given by:

$$A_1 = A_0 \cos(\beta - \gamma_1) \qquad \text{VIII - 1}$$

The corresponding energy measurable at the exit of the analyzer becomes

$$E_1 = \left(\frac{A_0^2}{2}\right) \cos^2(\beta - \gamma_1) \qquad \text{VIII - 2}$$

However the presence of unpolarized light energy modifies the Malus's law since half the unpolarized energy E_u passes through the analyzer. Then the total energy reaching the light transducer is given by:

$$E_1 = \left(\frac{A_0^2}{2}\right) \cos^2(\beta - \gamma_1) + \frac{E_u}{2} \qquad \text{VIII - 3}$$

FIGURE VIII-1

COMPONENTS OF PLANE POLARIZED LIGHT IN THE DIFFERENCE TO THE SUN RATIO

-OA_0 amplitude of plane polarized light at the angular distance β from the reference axis ox

-OA_1 and OA_2 projections of the plane polarized amplitude A_0 onto the directions OP_1 and OP_2 of the axes of the analyzers at the angular distance γ_1 and γ_2 from the axis of reference.

γ_0 angular distance of the bissectrix OB of the angle $\gamma_2 - \gamma_1 = 2\theta$ from the direction ox.

α angular distance of the plane polarized amplitude OA_0 from the bissectrix OB

FIGURE VIII-2

POLARIZED AND UNPOLARIZED COMPONENTS OF THE ZODIACAL LIGHT

ox, oy orthogonal axes in the plane of the ecliptic

oz axis normal to the ecliptic.

V instantaneous position of the vehicle in the plane xoy

VT line of sight issued from the vehicle and at the angular elongation from the sun at the origin of the axes of reference.

OR radial direction issued from the sun and intersecting the line of sight VT

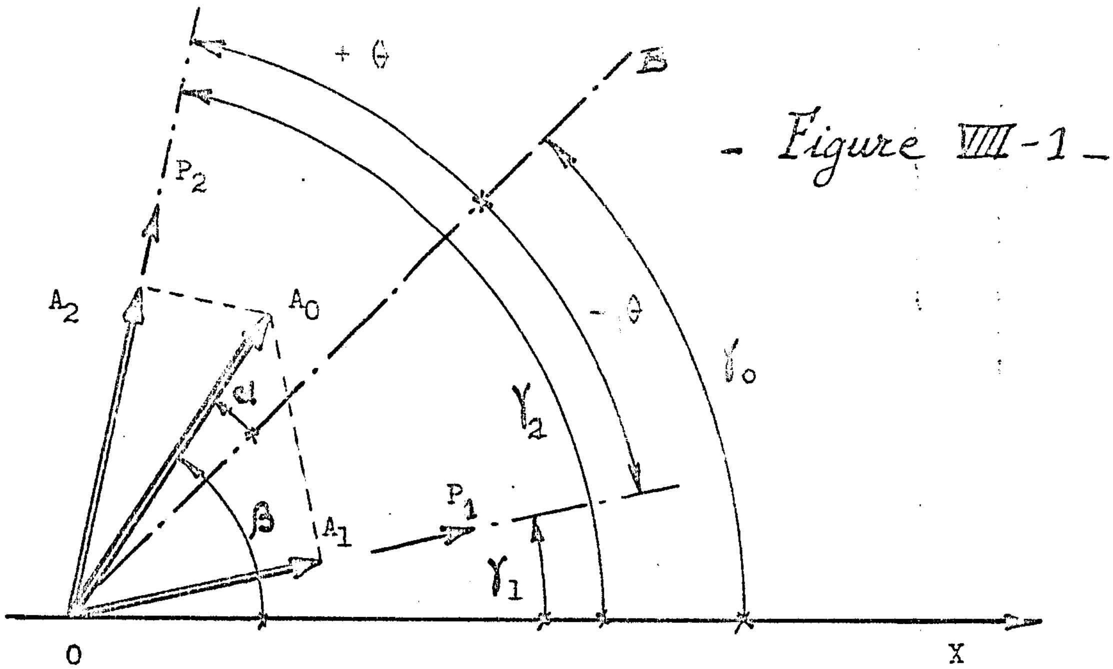
λ angular elevation of the direction OR reckoned from the plane of the ecliptic.

A_0 amplitude of plane polarized light

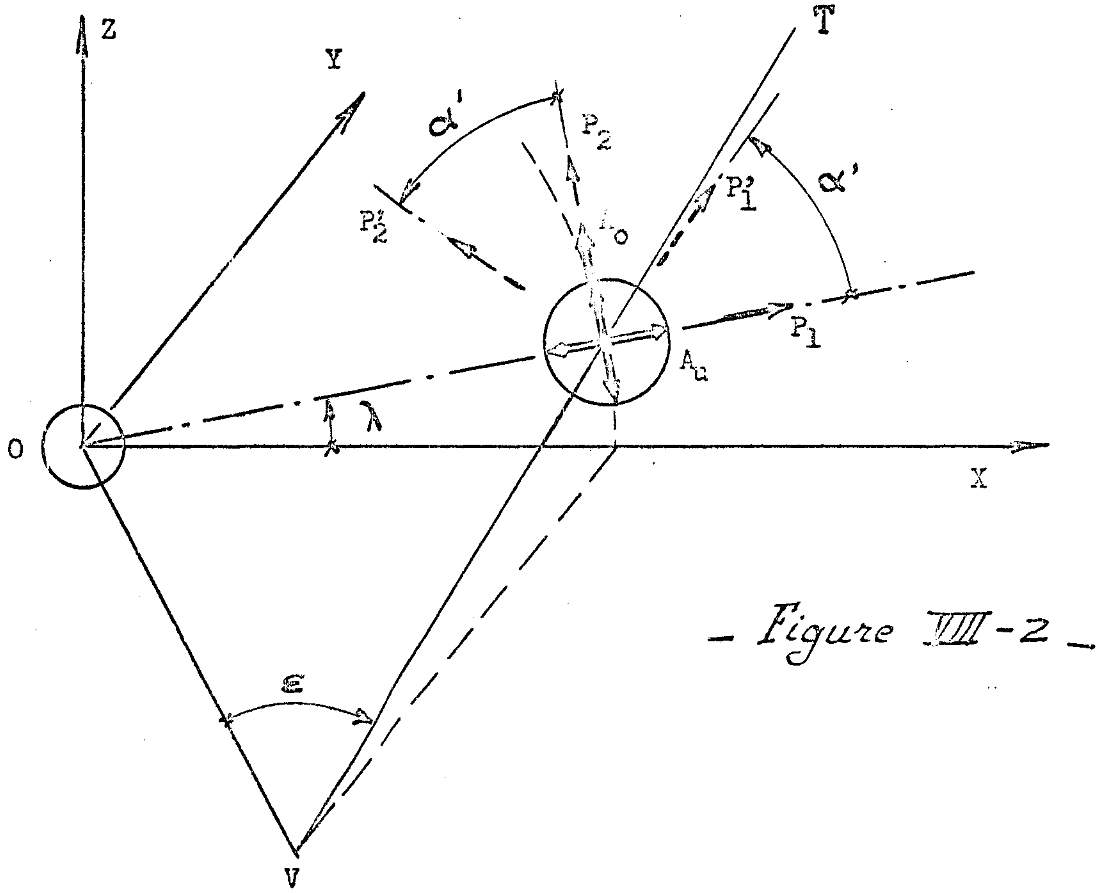
A_u equivalent amplitude of the unpolarized light

P_1 and P_2 directions of the analyser axis, parallel and orthogonal to the direction OR

P_1^2 and P_2^2 directions of the analyser axis consecutive to an angular displacement α'



Components of plane polarized light inthe
Difference to the Sum ratio



Polarized and unpolarized components of the zodiacal light

It is clear that a single measurement cannot determine the percentage of polarized light. Three unknowns are involved: $\left(\frac{A_o^2}{2}\right)$, β and E_u.

At least a second measurement at a different and known angular distance becomes necessary. In doing so one obtains the relationship:

$$E_2 = \left(\frac{A_o^2}{2}\right)\cos^2(\beta - \gamma_2) + \frac{E_u}{2} \qquad \text{VIII - 4}$$

If one wants to solve for the three unknowns separately, a third angular position γ_3 will be required. However, the kind of informations which can be obtained with only two angular positions must be examined.

Making use of the difference to the sum method of polarimetric analysis, one obtains the difference:

$$E_2 - E_1 = \left(\frac{A_o^2}{2}\right)\left[\cos^2(\beta-\gamma_2) - \cos^2(\beta-\gamma_1)\right] \qquad \text{VIII - 5}$$

in which the unpolarized light energy E_u is missing. On the contrary, the sum of the measured energies yields

$$E_2 + E_1 = \left(\frac{A_o^2}{2}\right)\left[\cos^2(\beta-\gamma_2) + \cos^2(\beta-\gamma_1)\right] + E_u \qquad \text{VIII - 6}$$

Hence one obtains the ratio R_α as per:

$$R_\alpha = \frac{E_2-E_1}{E_2+E_1} = \frac{\left(\frac{A_o^2}{2}\right)\left[\cos^2(\beta-\gamma_2) - \cos^2(\beta-\gamma_1)\right]}{\left(\frac{A_o^2}{2}\right)\left[\cos^2(\beta-\gamma_2) + \cos^2(\beta-\gamma_1)\right] + E_u} \qquad \text{VIII - 7}$$

This expression can be written in a simpler form. The angles γ_1 and γ_2 can be written in terms of their half sum which is the angular distance γ_o of the bissectrix of the angle $\gamma_2 - \gamma_1$, existing between the two analyzer positions, from the reckoning axis.

$$\gamma_2 + \gamma_1 = 2\gamma_o \qquad \text{VIII - 8}$$

$$\gamma_2 - \gamma_1 = 2\theta \qquad \text{VIII - 9}$$

FIGURE VIII-[illegible]

TYPICAL SCAN OF THE ZODIACAL LIGHT

ox, oy orthogonal axes of reference in the plane of the ecliptic

oz normal to the ecliptic

V position of the vehicle whose axis of spin VC is contained in the plane of the ecliptic and at the elongation $(\varepsilon_1 + \varepsilon_2)/2$ from the sun O.

VT instantaneous position of the line of sight at the angular distance φ from the spin axis VC.

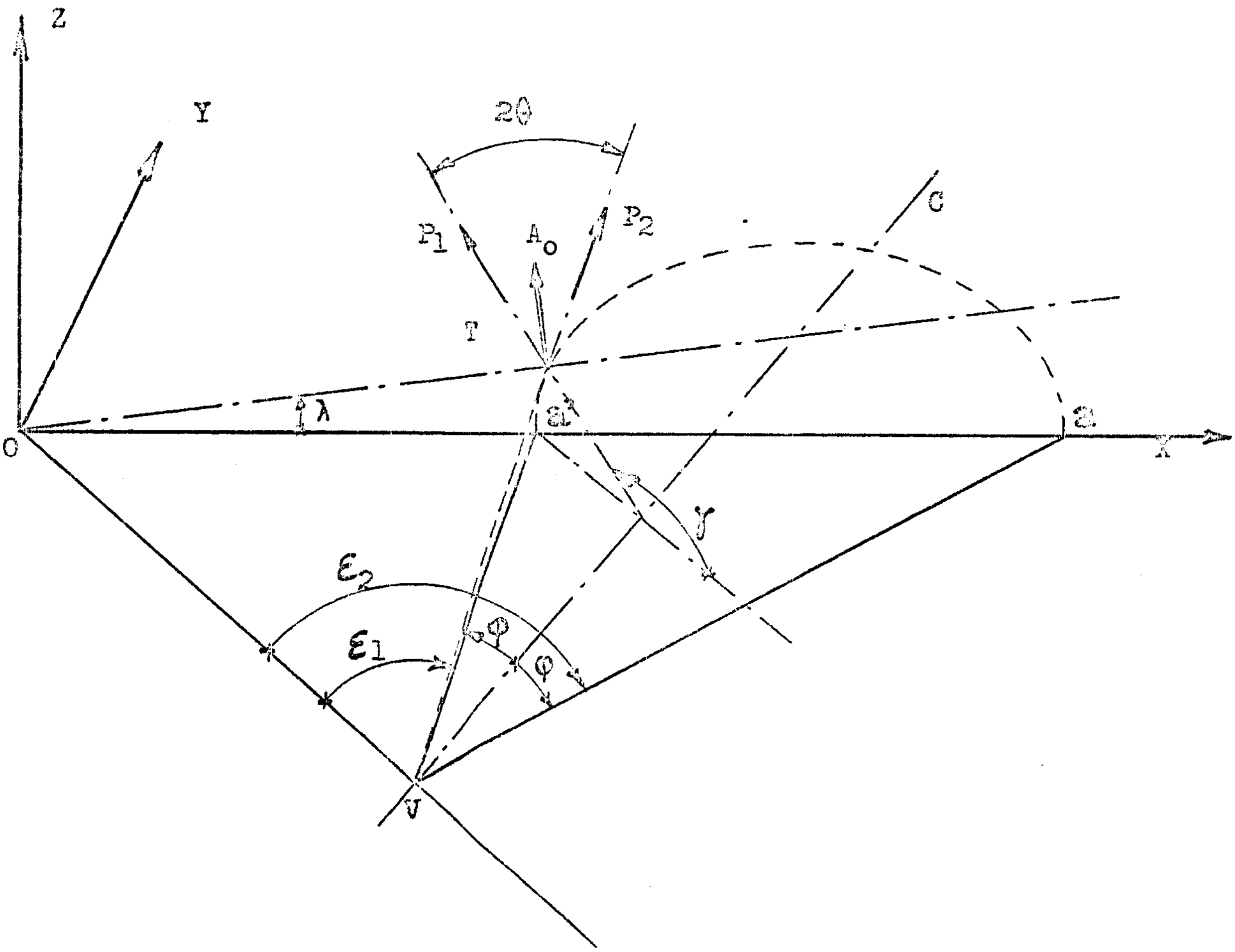
aa' the intersections of the trace of the scan-dash curve- in the plane XCZ

OT direction of the radius issued from the sun and intersecting the line of sight in T

A_o amplitude of the polarized light, normal to the direction OT

TP_1 and TP_2 directions of the analysers axes making the elected angle 2θ among themselves

_ Figure VIII - 3 _



Typical scan of the zodiacal light

Hence the values

$$\gamma_2 = \gamma_0 + \theta \qquad \text{VIII-10}$$

$$\gamma_1 = \gamma_0 - \theta \qquad \text{VIII-11}$$

Consecutively, the angular distances $\beta-\gamma_2$ and $\beta-\gamma_1$ can be written as per:

$$\beta-\gamma_2 = \beta-\gamma_0-\theta = \alpha-\theta \qquad \text{VIII-12}$$

$$\beta-\gamma_1 = \beta-\gamma_0+\theta = \alpha+\theta \qquad \text{VIII-13}$$

The angular difference $\beta-\gamma_0$ measures the angular distance α of the direction of the plane polarized amplitude with respect to the direction of the OB mean angular distance γ_0 of the analyzer axes.

Introducing those angular equivalences into the equation VIII-7 one gets, after simplification:

$$R_\alpha = \frac{E_2-E_1}{E_2+E_1} = \frac{\left(\frac{A_0^2}{2}\right)\sin 2\theta \sin 2\alpha}{\left(\frac{A_0^2}{2}\right)\left[1+\cos 2\theta \cos 2\alpha\right] + E_u} \qquad \text{VIII-14}$$

This expression is sign sensitive with respect to the angular distance α to the mean direction OB. Yet its denominator takes an invariant form when:

$$\cos 2\theta = 0 \quad \therefore \quad \theta = \pi/4 \qquad \text{VIII-15}$$

and measures the sum of the polarized and unpolarized energies:

$$\frac{A_0^2}{2} + E_u \qquad \text{VIII-16}$$

which is the total energy of reference for the degree of polarization p.

But under this condition where $\theta = \pi/4$, the value of $\sin 2\theta$ equals 1 and the difference to the sum ratio becomes

$$R_\alpha = \frac{\left(\frac{A_0^2}{2}\right)\sin(2\alpha)}{\left(\frac{A_0^2}{2}\right) + E_u} \qquad \text{VIII-17}$$

Being equal to zero for $\alpha = 0$, the ratio R_α passes through the maximum value :

$$R_{\alpha_0} = \frac{\left(\frac{A_0^2}{2}\right)}{\left(\frac{A_0^2}{2}\right) + E_u} \qquad \text{VIII - 18}$$

for $\alpha = \pi/4$ and measures therefore the degree of polarization. The conditions of measurement are clearly defined by:

$$\alpha = \pi/4 = \beta - \gamma_0 \qquad or \qquad \beta = \gamma_0 + \pi/4$$

$$\gamma_2 = \gamma_0 + \pi/4 \qquad \beta - \gamma_2 = 0$$

$$\gamma_1 = \gamma_0 - \pi/4 \qquad \beta - \gamma_1 = \pi/2$$

Thus for that particular case, the direction of the plane polarized amplitude is parallel to the direction OP_2 and normal to OP_1.

Considering the measurement of the degree of polarization of the zodiacal light, in that case, the direction of the plane polarized amplitude is known to be normal to the ray issued from the sun. Thus, if one orientates one of the analyzer axis normal to the radial direction R and the other parallel to it, the accessible difference to the sum ratio measures directly the degree of polarization of the zodiacal light. Only two measurements appear to be required.

Attainable accuracies and limits of uncertainties are functions of the instrumental resolving power ΔE in energies detection as well as of the degree of definition in θ and α.

Achievable accuracies and uncertainties can be expressed analytically for discussion. One cannot consider taking the total differential of the ratio R_α as a function of both θ and α. An indetermination would result consecutively to the fact that both $\cos(2\theta)$ and $\cos(2\alpha)$ are equal to zero in that particular case. One must resort to the incremental computation of ΔR_α in R_α consecutive to the incremental variations $\Delta\theta$ and $\Delta\alpha$.

This operation leads to the expression:

$$\Delta R_{\alpha} = -2 \frac{\left(\frac{\eta_0}{2}\right)^2}{\left(\frac{\eta_0}{2}\right)^2 + E_u} \left[(\Delta\alpha)^2 + (\Delta\theta)^2 \right] \qquad \text{VIII-19}$$

in which the variations $(\Delta\alpha)$ and $(\Delta\theta)$ appear as ab[illegible]ing terms of the second order.

Consecutively the relative indetermination δR_{α} in R_{α} amounts to:

$$\frac{\delta R_{\alpha}}{R_{\alpha}} = -2(\delta\alpha)^2 - 2(\delta\theta)^2 \qquad \text{VIII-20}$$

The percentage error consecutive to $\delta\theta$ can be neglected since relative positioning of the axes of the analyzers can be achieved at least to 1/100 of a degree yielding a value in $(\delta\theta)^2$ of the order of:

$$\left(\frac{\delta R_{\alpha}}{R_{\alpha}}\right)_{\alpha} \simeq -2 \times 3.05 \times 10^{-8} \qquad \text{VIII-21}$$

but the uncertainty in $\delta\alpha$ cannot be dismissed. The actual trace of the scanning can intersect the plane of the ecliptic at angles deviating from 90° by several degrees, reflecting the same amount of deviation in the relative angular direction of the plane polarized amplitude.

It seems realistic to admit a possible error of $\Delta\alpha = 3°$ which, in turn, leads to:

$$\left(\frac{\delta R_{\alpha}}{R_{\alpha}}\right)_{\theta} \cong -2 \times 2.704 \times 10^{-3} \qquad \text{VIII-22}$$

or a deviation of some 0.541%.

However, the undetermination $\Delta\alpha$ in α can be lifted. Indeed one can impose a known rotation α' to the direction of the plane polarized light by means of a quartz plate for instance. This is equivalent to rotate the axes of the analyzers by the angular distance $-\alpha'$. Thus a new difference to sum ratio R'_α is found as:

$$R'_\alpha = \frac{\left(\frac{A_o^2}{2}\right)\sin 2\theta \sin 2(\alpha+\alpha')}{\left(\frac{A_o^2}{2}\right)\left[1+\cos 2\theta \cos 2(\alpha+\alpha')\right]+E_u} \qquad \text{VIII - 23}$$

The denominator is not affected since $\cos 2\theta$ remains equal to zero. But the numerator can be changed at will and particularly made equal to 0. If then α' is chosen to be equal in absolute value to α but opposite in sign

$$\alpha' + \alpha = 0 \qquad \text{VIII - 24}$$

then the ratio R'_α as obtained will measure the departure $\Delta\alpha$ of α from its assigned value as per:

$$R'_\alpha = \frac{\left(\frac{A_o^2}{2}\right)\sin(2\Delta\alpha)}{\left(\frac{A_o^2}{2}\right)+E_u} \qquad \text{VIII - 25}$$

for $\theta = \pi/4$

Note that the sum $R_\alpha + R'_\alpha$ cannot cancel the deviation $\Delta\alpha$ since the value of this sum is

$$R_\alpha + R'_\alpha = \frac{\left(\frac{A_o^2}{2}\right)}{\frac{A_o^2}{2}+E_u} \times \left[1 - 2(\Delta\alpha)^2 + 2\Delta\alpha\right] \qquad \text{VIII - 26}$$

Upon examination of the expression for R'_α it is found that it is equal and opposite in sign to half the derivative of R_α and takes full significance from that property. Indeed the difference of the ratio R_α can be written:

$$\left(\frac{A_o^2}{2}\right)\sin 2(\alpha+\Delta\alpha)$$

and for $\alpha = \pi/4$ it takes the value

$$\left(\frac{A_o^2}{2}\right)\sin\left(\frac{\pi}{2} + 2\Delta\alpha\right) = \left(\frac{A_o^2}{2}\right)\cos\left(2\Delta\alpha\right) \qquad \text{VIII-27}$$

The derivative of the sum amounts to :

$$\left(E_2 + E_1\right)' = -\frac{1}{2}\left(\frac{A_o^2}{2}\right)\sin\left(2\Delta\alpha\right) \qquad \text{VIII-28}$$

Also between the measurements with $\alpha = +\pi/4$ and $\alpha' = -\pi/4$ one finds the relationship

$$\frac{R'_\alpha}{R_\alpha} = \frac{\sin(2\Delta\alpha)}{\cos(2\Delta\alpha)} = \tan\left(2\Delta\alpha\right) \qquad \text{VIII-29}$$

which determines $\Delta\alpha$ unequivocally.

Therefore within the range of $\Delta\alpha$ smaller than 10°, the exact value of the degree of polarization P_o is accessible through the relationship.

$$P_o \cong \frac{R_\alpha}{1 - \frac{1}{2}\left(\frac{R'_\alpha}{R_\alpha}\right)^2} \cong R_\alpha\left[1 + \frac{1}{2}\left(\frac{R'_\alpha}{R_\alpha}\right)^2\right] \qquad \text{VIII-30}$$

if one considers that for small angle $\Delta\alpha$ one can write :

$$\tan\left(2\Delta\alpha\right) \cong 2\Delta\alpha$$

and that $\left[R'_\alpha/R_\alpha\right]^4$ is negligible for all purposes.

Also, it has been shown that the difference to the sum ratio R_α leads, around the zero origin, to the uncertainty:

$$\delta R'_\alpha = \frac{\delta E}{E} = 2\tan\theta \times \delta(\Delta\alpha) \qquad \text{VIII-31}$$

hence $\Delta\alpha$ can be determined within an uncertainty of :

$$\delta(\Delta\alpha) = \frac{1}{2\tan\theta} \times \frac{\delta E}{E} \qquad \text{VIII} - 32$$

wherein $\delta E/E$ expresses the uncertainty in the measurement of the energies. Taking $\delta E/E = 0.001$ as found in an analog system and $\theta = \pi/4$ the angle $\Delta\alpha$ becomes available at :

$$\delta(\Delta\alpha) = 0.000\,5 \text{ radian} \doteq 0.028\,6 \text{ degree}$$

From the analytical treatment it appears that the method of measurement possesses both the range and the accuracy compatible with the problem of determination of the degree of polarization of the zodiacal light.

It has another advantage. The sum of the energies $E_2 + E_1$ or $E_2' + E_1'$ an invariant, can be compared against the irradiance of the sun through a calibrated attenuator by means of peak rider technique.

Owing to the distribution of the size of the particles and of their nature, a certain percentage of ellipticity could be expected. The ellipticity associated with the observable plane polarized energy cannot be detected directly by the use of analyzers alone. Half of the circular polarized energy present passes through the analyzer irrelevant of the orientation of its axis. This energy, undistinguishable from the unpolarized light, if present, is therefore integrated as unpolarized.

A general analytical relationship for the case of ellipticity has been established for the energy emerging from the analyzer as

$$E = 2\left(\frac{A_0}{2}\right)^2 e^{-(R+L)} \times \cos^2\alpha + \frac{1}{2}\left(\frac{A_0}{2}\right)^2 \times \frac{\left(e^{-R} - e^{-L}\right)^2}{e^{-(R+L)}} \qquad \text{VIII} - 33$$

The right and left components forming the elliptically polarized light are taken as

$$\left(\frac{A_o}{2}\right)_x e^{-R} \quad \text{and} \quad \left(\frac{A_o}{2}\right)_x e^{-L}$$

for the purpose of generalization. Here, the angle α defines the angular position of the major axis of the elliptical trajectory of the resultant vector.

However, if one orientates the analyzer axis as to be orthogonal to the direction of the major axis of the ellipsa, giving $\alpha = \pi/2$, the plane polarized amplitude is extinguished. Under that condition, one can place ahead of the analyzer a quarter wave plate with its fast axis either ahead or behind the direction of the plane polarized amplitude. In so doing, the energies emerging from the analyzer and detected by the light transducer becomes either

$$E_{c2} = \frac{1}{2}\left(A_o + b\right)^2 + E_u \qquad \text{VIII-34}$$

or

$$E_{c1} = \frac{1}{2}\left(A_o - b\right)^2 + E_u \qquad \text{VIII-35}$$

Applying the difference to the sum ratio method one obtains:

$$R_{de} = \frac{2 A_o b}{(A_o^2 + b^2) + E_u} \qquad \text{VIII-36}$$

Hence the percentage of circular polarized light becomes accessible. Experimentation to evaluate the possibility of this method will be performed soon in the case of the study of dichroism. From the observed behavior it will be possible to establish a definite analysis for the case of zodiacal light which differs appreciably from the dichroism where the unpolarized energy remains a very small percentage of the polarized one.

Chapter IX

SOLAR SPECTRAL IRRADIANCE and LIGHT TRANSDUCER PERFORMANCES v.s. BRIGHTNESS OF THE ZODIACAL LIGHT

In the search for appropriate light transducer and relate their characteristics to the problem of measurement of the brightness of the zodiacal light, difficulties were experienced on account of ambiguities and lack of definition of the energies of reference. Most of the manufacturers indicate the sensitivity in amper per lumen, though this unit of energy is related to the sensitivity of the eye which varies with the wave length of the light.

If one refers to the accepted least mechanical equivalent of light, there one finds the equivalence of 1.61×10^4 ergs x sec^{-1} at 5560 A°. Yet, this equivalence valid for the given wave length loses most of its meaning in view of the actual method of testing performed under white light illumination as produced by a tungsten light source maintained at 2870°K.

Hence the available data, in general, refer to the integral of spectral distribution times the spectral sensitivity of the transducer.

One could consider the definition of a lumen given by C. W. Allen (Astrophysical Quantities 1955): a lumen is the luminous flux from an area $1/60\ \pi$ cm^2 of a black body surface at the temperature of melting platinum, 2042° K.

This quantity of energy, as defined, appears to be the flux corresponding to the integrated spectral distribution of the emitted energy. Indeed computing the emitted energy by one cm^2 area of melted platinum at 2042° K for the wave length of $\lambda = 5300$ A°, one would obtain:

$$\Delta E = 1520 \text{ ergs}/cm^2 \cdot sec. \, A^\circ_o$$

and a lumen equivalence of

$$1\ lm = \frac{1520}{60\pi} = 8.05\ ergs/sec.A^{\circ}$$

a disproportionate figure

Hence the evaluation of the lumen on that basis requires the integration of the energy throughout the whole spectrum of emission of the source at 2042° K and correlating this value against that corresponding to a temperature of 6000° K, the approximate temperature of the sun. Yet this procedure would be quite cumbersome for the determination of the available energy in terms of wave length.

Considering those facts it appears to be more convenient and more reliable to compute the emission of energy per unit area at temperatures of 6000° K and 2870° K and to relate the available spectral relative sensitivity of the cathodes.

Using the Planck's formula

$$E = c_1 \lambda^{-5}\left[e^{\frac{c_2}{\lambda T}} - 1\right] \qquad \text{IX - 1}$$

with

$$c_1 = 2\pi h c^2 = 3.74 \times 10^{-5}\ ergs/cm^2.sec \qquad \text{IX - 2}$$

for λ and $\Delta\lambda$ in cm: C.G.S. units, and

$$c_2 = \frac{hc}{K} = 1.438\ \ cm.\ degree \qquad \text{IX - 3}$$

the emitted energy E has been computed at different wave lengths for an interval $\Delta\lambda$ = 100 A° and for the temperatures of 6000° K and 2870° K. These values are given in the following chart as well as the corresponding brightness of the emitting surface as given by

$$B = E/\pi \qquad \text{IX - 4}$$

Energy Emitted By A Black Body

in ergs / cm^2. sec. 100 A°

Wavelength	Temperature 6000° K		Temperature 2870° K	
λ in A°	Emitted Energy E_{6000}	Brightness $B = E/\pi$	Emitted Energy E_{2870}	Brightness $B = E/\pi$
9000	476×10^6	151.5×10^6	24.32×10^6	7.74×10^6
8000	603×10^6	192×10^6	21.97×10^6	6.98×10^6
7000	751×10^6	239×10^6	17.3×10^6	5.5×10^6
6500	833×10^6	265×10^6	14.5×10^6	4.62×10^6
6000	905×10^6	288×10^6	11.5×10^6	3.66×10^6
5500	965×10^6	307×10^6	8.25×10^6	2.62×10^6
5300	987×10^6	314×10^6	7.01×10^6	2.23×10^6
5000	1005×10^6	334×10^6	5.32×10^6	1.69×10^6
4500	930×10^6	296×10^6	3.00×10^6	0.95×10^6
4000	915×10^6	291×10^6	1.23×10^6	0.391×10^6
3500	759×10^6	241×10^6	0.436×10^6	0.139×10^6
3000	525×10^6	167×10^6	0.086×10^6	0.027×10^6
2500	260×10^6	82.7×10^6	7.73×10^3	2.46×10^3
2000	74×10^6	23.5×10^6	0.155×10^3	0.049×10^3

Those values are presented in form of a graph for rapid examination.

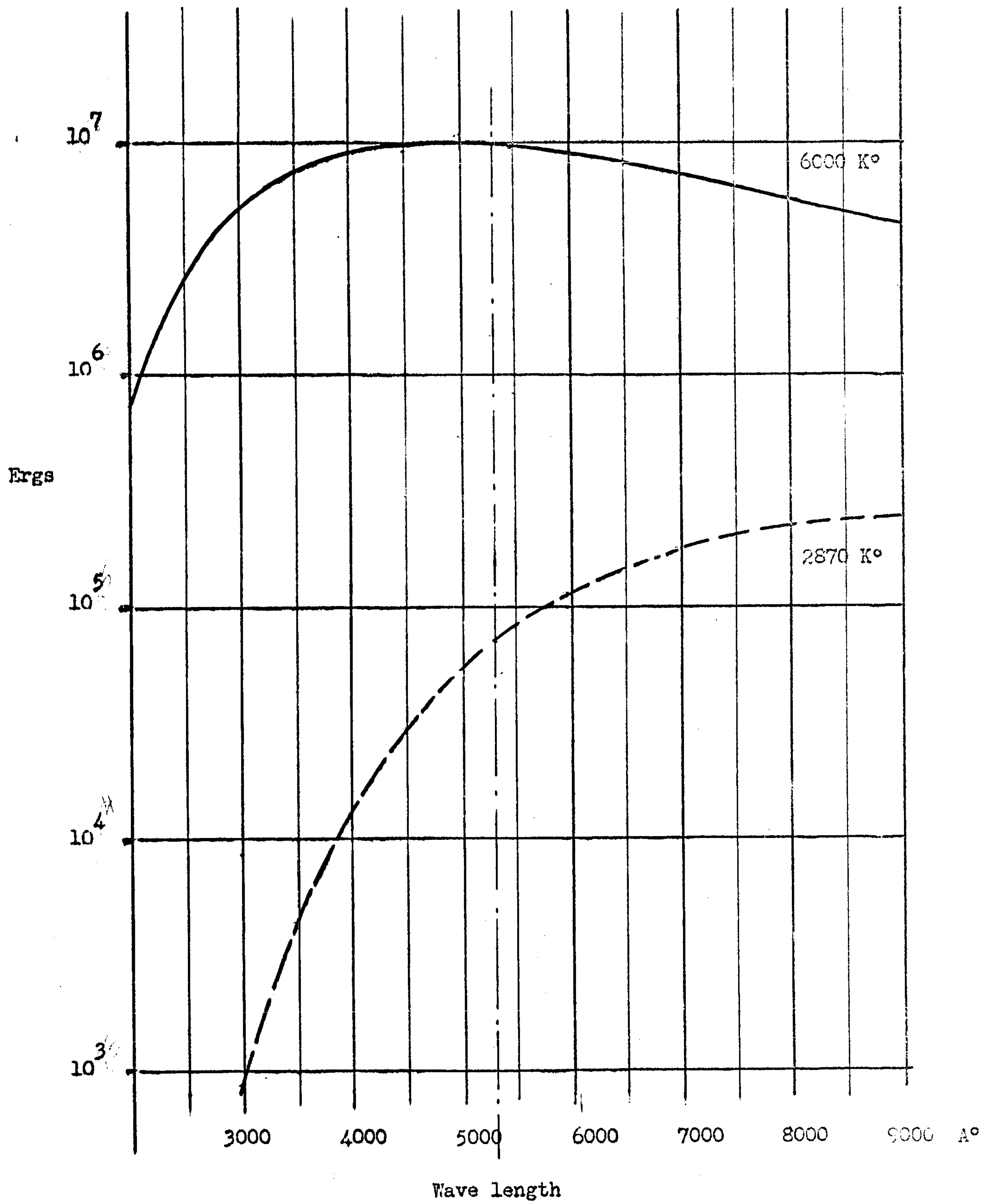
FIGURE IX - 1

EMITTED ENERGY IN ERGS/cm^2. sec. A^o v.s. WAVE LENGTH

The energy emitted by a perfect diffusor for the temperatures of 6000^{o}K and 2870^{o}K, as computed from the Planck's formula, are plotted in terms of the wave length.

Figure IX-1

Emitted energy in ergs / cm2;sec; A°



The values of the wave lengths corresponding to the peak of emission are obtained from Wien's law

$$\lambda_p \cong 0.2897 / T \qquad \text{IX-5}$$

and are found to be

$$\lambda_p \cong 4840 \text{ A}^\circ \quad \text{for} \quad T = 6000^\circ K$$

$$\lambda_p \cong 10100 \text{ A}^\circ \quad \text{"} \quad T = 2870^\circ K$$

It is seen that the brightness of 314×10^6 ergs/cm^2. sec.100 A$^\circ$ at $\lambda = 5300$ A$^\circ$ for the temperature of 6000° K differs from the compilation by C. W. Allen (1955) for the sun brightness, at the same wave length, reported at 294×10^6 ergs/cm^2. sec.100 A$^\circ$.

The relative difference amounts to

$$\frac{dE}{E} = \frac{+20}{294} \cong +0.068$$

Considering the differential of the Planck's formula with respect to the temperature, that is

$$\frac{dE}{E} \cong \left(\frac{c_2}{\lambda T}\right) \frac{dT}{T} \qquad \text{IX-6}$$

one obtains a temperature difference of some:

$$\Delta T \cong 80^\circ K$$

corresponding to 5920° K for the equivalent temperature of the sun averaged over its disk. Hence the evaluation of the performance of the light transducer, the temperature of 6000° K has been elected. The correction in ΔE, amounting to a small percentage, can be easily introduced.

The irradiant energies per cm^2. sec.$\Delta\lambda$ are obtained directly by dividing the computed emitted energies by:

$$\left(1 \text{ A.U.}/R_\odot \right)^2 = 4.62 \times 10^4$$

the square of the ratio of the distance of the earth to the sun by the radius of the sun.

However to permit a direct comparison with the values of Johnson's (1954) Solar Spectral Irradiance given in ergs/cm^2. sec. A^o at the distance of the earth. The emitted energies already computed for $\Delta\lambda$ = 100 A^o are divided by 4.62 x 10^6 instead and reported in the next chart.

Those data can be used to determine directly the reference anode current output of the photomultiplier. The reference current equals the product of the irradiant energies per cm^2. sec. A^o times the cathode radiant sensitivity σ_K in amper/watt, times the elected amplification factor . Those values have been determined for the case of the E.M.R. 541A-01-14 which, from the preliminary search, stands out for its extremely low dark current and noise equivalent current input.

The reference anode currents are also entered in the following chart for rapid reference and given in the corresponding graph in terms of wave length.

Irradiant Energy E_{6000} in Ergs, cm^{-2}, sec^{-1}, A^{o-1}

Reference Anode Current in Amp for E.M.R. 541A-01-14

v.s. Wavelength for T = 6000^{o} K

Wavelength λ A^{o}	Irradiant Energy E ergs/cm^2, sec, A^{o}	Cathode Radiant Sensitivity σ_K A/W	Anode Current I_{ac} in Amp at Amplification G = 10^6
8000	130 ergs		
7000	163		
6500	180	0.0012 A/W	0.022 A
6000	196	0.0092	0.180
5500	209	0.0327	0.684
5300	213	0.0446	0.950
5000	217	0.0477	1.035
4500	201	0.054	1.09
4000	198	0.0492	0.975
3500	164	0.0331	0.54
3000	114		

FIGURE IX - 2

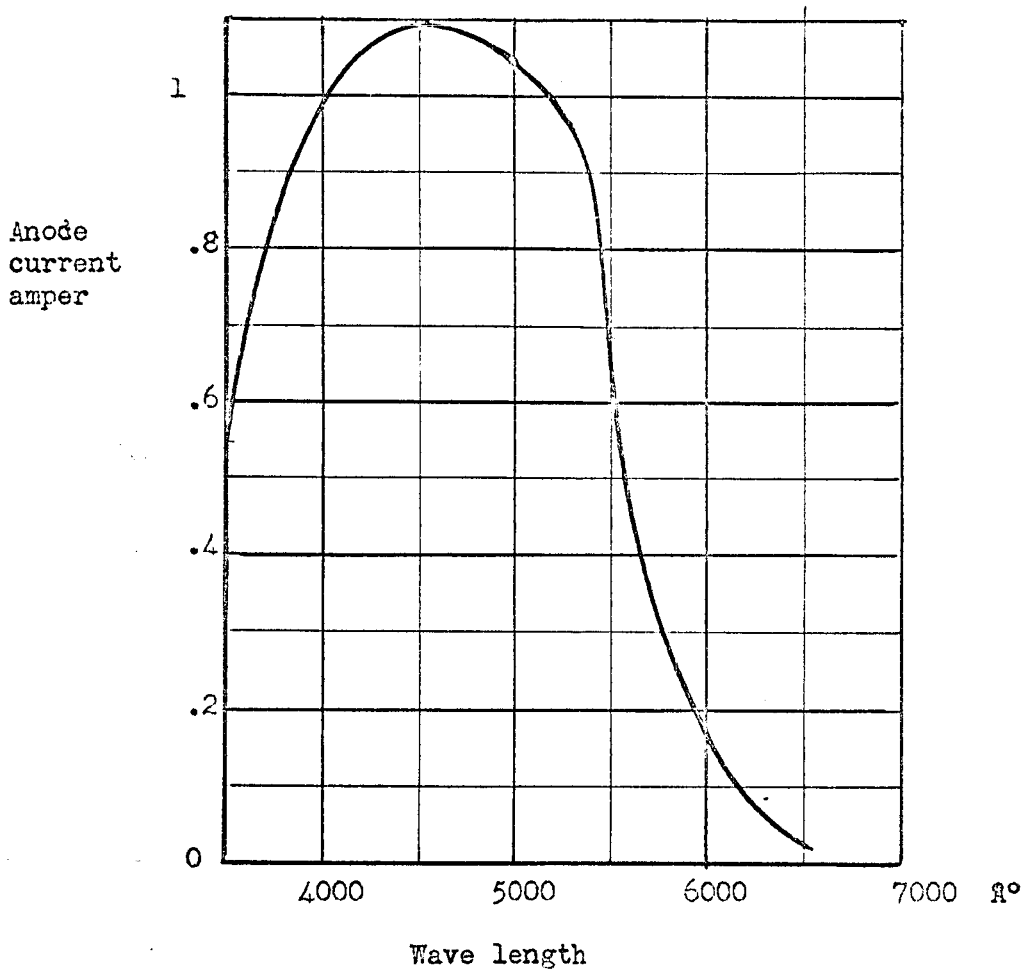
TYPICAL ANODE CURRENT - AMPERS AT AN AMPLIFICATION
$G=10^6$ FOR SOLAR ENERGY /cm^2. sec. A° AT THE DISTANCE
OF THE EARTH

The anode current output available from a typical E.M.R. 541A-01-14 Photo multiplier in function of the solar irradiant energy, at the distance of the earth, computed on the basis of a 6000°K temperature, is shown as reference by the graph in terms of wave length.

Actual solar irradiant energies can be evaluated in terms of per cent deviation from the basis of reference.

– Figure IX-2 –

Typical anode current-Ampers- at an amplification $G = 10^6$ for solar energy / cm^2. sec. A° at the distance of the earth.



One can compare the irradiant energies, computed for T = 6000° K, against some of the solar irradiances, above the atmosphere of the earth, indicated by Johnson as indicated in the following chart.

Irradiances: ergs/cm^2 x sec x A°

Computed at T = 6000° and reported (Johnson) 1954

Wavelength λ	5000 A°	5300	5500	6000 A°
Irradiances: Computed	217 ergs	213	209	196
Irradiances: Reported	198 ergs	195	195	181
% Deviation	+ 9.6%	+ 9.23%	+ 7.2%	+ 8.3%

It is noted that the deviations are consistant and that the values obtained at 6000° K should be decreased by an average of 8.5% for final designing.

In view of the agreement existing between reported and computed values of the solar irradiances one can safely predict the current output of the photomultiplier in function of the brightness $\mathcal{B}_z$ of the zodiacal light which has been referred to the brightness $\bar{\mathcal{B}}_\odot$ of the integrated solar disc. But the zodiacal light being an extended source, the available flux of energy per unit area becomes equal to the product of the brightness of the source times the solid angle in steradian, corresponding to the elected angular field of view U of the instrument.

Normally in instrument design, the field of view is expressed in degree. Its corresponding solid angle can be taken as

$$\Delta\Omega_U \cong \pi\left(\frac{\pi U}{360^\circ}\right)^2 \cong \pi \times 0.761 \times 10^{-4}\, U^2 \qquad \text{IX-7}$$

without sensible error for the range of few degrees in U.

Thus the flux of energy per $cm^2 \times sec \times A^{o}$ due to the zodiacal light is given by either

$$E_{Ze} = \bar{B}_{\odot} \left(\frac{B_Z}{\bar{B}_{\odot}} \right) \pi \times 0.761 \times 10^{-4} U^2 \qquad IX-8$$

or

$$E_{Ze} = \bar{E}_{\odot} \left(\frac{B_Z}{\bar{B}_{\odot}} \right) \times 0.761 \times 10^{-4} U^2 \qquad IX-9$$

However since there is correspondence between spectral irradiances $(E_e)_{6000^\circ}$ computed for the temperature of 6000° K and the values reported by Johnson (1954) one can make use of the relationship:

$$E_{Ze} = E_e \left(\frac{B}{\bar{B}_{\odot}} \right) \times \left(\frac{U}{U_{\odot}} \right)^2 \qquad IX-10$$

Where $U_{\odot}$ is the angular diameter of the sun taken at 32'. This leads to the relationship:

$$E_{Ze} = E_e \left(\frac{B}{\bar{B}_{\odot}} \right) \times 3.52 \times U^2 \qquad IX-11$$

from which one obtains directly the energy collected by the required afocal telescope characterized by its equivalent clear diameter D and its over all transmission factor T_t.

Hence the relationship: IX-12

$$\Sigma E_{Ze} = E_e \left(\frac{B}{\bar{B}_{\odot}} \right) \times 3.52 \times \frac{\pi}{4} \times T_t \times D^2 U^2 = E_e \left(\frac{B}{\bar{B}_{\odot}} \right) \times 2.767 \times T_t \times D^2 U^2$$

for the flux of energy available at the cathode of the light transducer.

From this last relationship it is seen that the available energy at the cathode of the transducer varies with the square of the angle of field of view U and the square of the clear diameter of the afocal telescope. At first glance, practical dimensions of D = 10 cm and U = 2° would give a collecting power $\mathcal{J}^2 U^2$ or equivalent surface of some

$$S_c = 2.6767 \times 100 \times 4 \cong 1071 \ cm^2$$

leading to a high signal to noise ratio.

But, one faces serious limitations imposed by the inherent behavior of an afocal telescope be it of the positive-positive type, as shown in the figure, or of the positive-negative type.

First of all and most important in the discussion appears to be the angular degree of collimation U' at the exit pupil which is located at the principal plane of the secondary objective. It is expressed in terms of the angular field of view U of the system and of the focal lengths f and f' of the primary and secondary objectives by:

$$U' \cong \frac{d_f}{f'} \cong U \times \frac{f}{f'} \qquad \text{IX - 13}$$

since the angular field of view remains small, a few degrees: the angle equals its tangent for practical purpose.

The degree of collimation, measured by U', must be kept within the practical angular acceptance of the polarizing element which remains always limited by consideration of total extinction of the field.

Certainly polarizers elements can be selected which admit rather large angles of collimation or field of view such as: Polaroid film or Nicol's prisms but then total extinction cannot be secured. On the other hand prisms such as the Glan's or the Rouy's prisms achieve total extinction but their total field of view is limited to some 8 degrees. Also the transmission of the polarizing element must be considered carefully. The angular acceptance of the prism must be evaluated also with respect to the spherical aberrations of both primary and secondary objectives which restrict somewhat the angle of field of view depending upon the residual caustic.

FIGURE IX - 3

A FOCAL TELESCOPE - GEOMETRIC PARAMETERS

D clear diameter of the front objective

f focal length of the front objective

f' focal length of the secondary objective

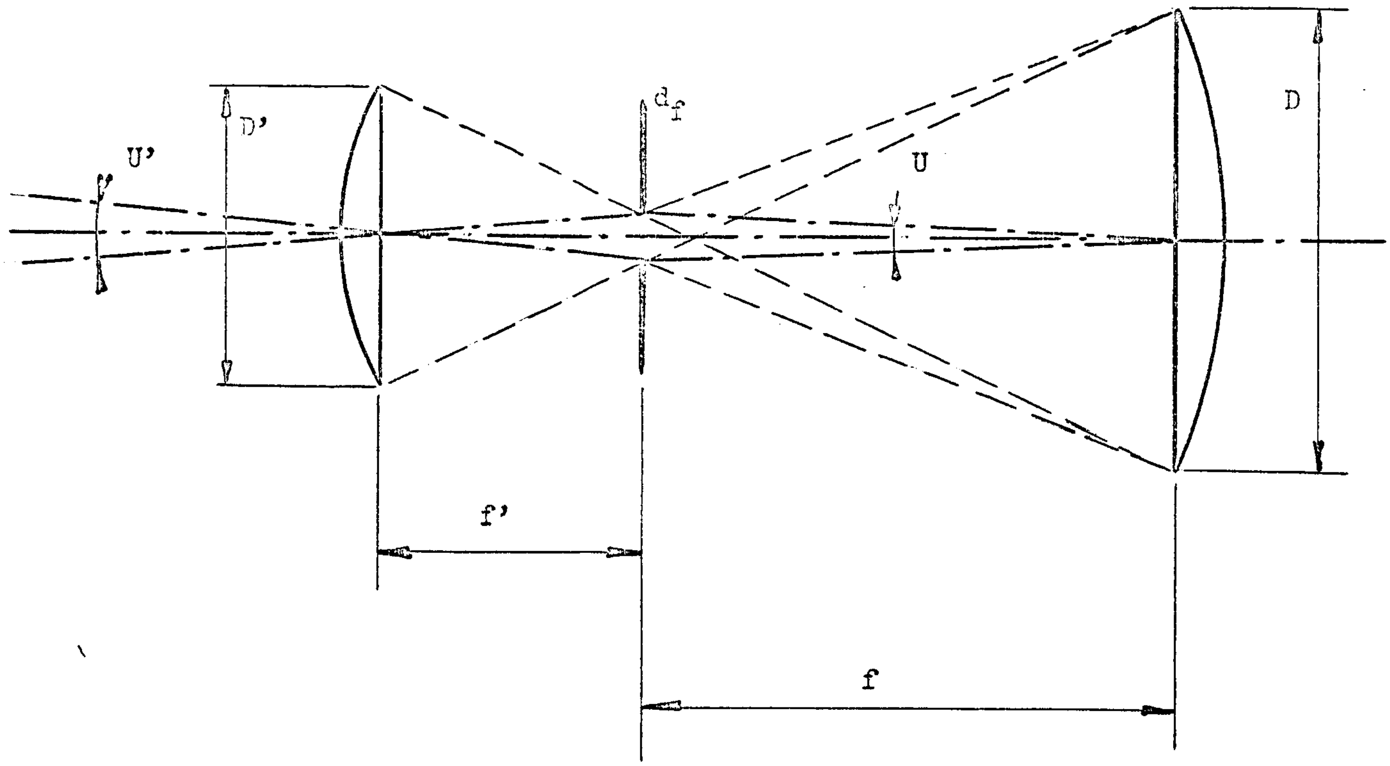
D' clear diameter of the secondary objective

U angular field of view for the front objective limited by the aperture d_f of the field diaphragm

U' relative field of view for the secondary objective defined by the aperture d_f of the diaphragm

- Figure IX -3

Afocal telescope - Geometric parameters



A second factor which cannot be disregarded in the evaluation is the diameter D' of the collimated beam governed by the relative aperture of the afocal telescope. In fact these parameters are related to each other by

$$D' \simeq D \times \frac{f'}{f} \simeq D \times \frac{U}{U'} \qquad \text{IX - 14}$$

The exact optimum values will have to be determined in function of the restrictions of weight and volumes.

Keeping in mind the realization of the system one can take a practical value of $U' = 6^\circ$ for the collimation and a clear diameter of 6 cms for the primary objective. This choice of dimensions permits an angle of field of view $U = 2^\circ$. Hence an attainable collecting coefficient of

$$S = 2.767 \times (6)^2 \times (2)^2 = 398 \quad cm^2 \times d^{\circ 2}$$

Then the anode current output for a typical E.M.R. 541A-01-14 can be computed on the basis of a band pass $\Delta\lambda$ equivalent to 100 A°. Its expression being of the form:

$$I_a = \sigma_k \times 10^{-7} \times G \times 2.767 \times T_t D^2 U^2 \times E_e \left(\frac{B}{\bar{B}_\odot}\right) \Delta\lambda \times T_p \qquad \text{IX - 15}$$

Where :

σ_k	=	cathode sensitivity	=	A/Watt
G	=	amplification	=	10^6
$T_t D^2$	=	equivalent clear collecting	=	36 cm^2
U	=	angle of field of view	=	2°
E_e	=	irradiance	=	$ergs/cm^2 \times sec \times A^\circ$
$B/\bar{B}_\odot$	=	relative brightness of zodiacal light: minimum	=	1×10^{-13}
$\Delta\lambda$	=	spectral band pass	=	100 A°
T_p	=	transmission of the polarizer	=	45%

The transmission T_p for a polarizer varies greatly from one type to another. Polaroid film of good quality has an effective transmission of the order of 16%, Nicol's and Glan's prisms of the order of 23-to-25%. However the Rouy's prisms single and double cut, orthogonal to the "C" axis of the crystal have been tested at higher values without antireflection coating at the entrance and exit surfaces. Some experimental values are indicated here for reference. The transmissions are given for over all transmission of unpolarized and for polarized light.

Transmission for Rouy's Polarizing Prisms

	Single Cut		Double Cut	
Wavelength λ A°	Unpolarized Light	Polarized Light	Unpolarized Light	Polarized Light
6000	46 %	92 %	43.1 %	86 %
5500	46 %	92 %	42.5 %	85 %
5400	46 %	92 %	42.3 %	84.6 %
4500	45.4 %	90.8 %	42 %	84 %
4000	43.7 %	87.4 %	39.1 %	78.2 %
3500	42 %	84 %	36.5 %	73 %

Losses by reflection can be decreased considerably. A single magnesium fluoride coating, quarter wave length, reduces the reflection losses to only 1.2% per air to refracting material surface. Yet considerable progress has been achieved in this domain: J. T. Cox and G. Hass - J.U.S.A. - September 1962. For instance a triple coating defined by $\frac{\lambda}{4} MgF_2 + \frac{\lambda}{2} CeO_2 + \frac{\lambda}{4} CeF_3$ on the substrate secures a zero reflection loss at 5300 A° while the losses are still 0.4% at 4500 A° and 0.2% at 6000 A°.

The triple coating $\frac{\lambda}{4} MgF_2 + \frac{\lambda}{2} ZrO_2 + \frac{\lambda}{4} CeF_3$ on the substrate permits to achieve a range of some 3000 A° with a maximum loss by reflection of 0.3%.

Then it is safe to assign an over all transmission of 45% in unpolarized light for the double cut system and 49% for the single cut.

The over all transmission T_p = 45%, for unpolarized energy, is therefore taken as basis.

The values of the anode current I_a for different wave lengths as well as their ratio to the equivalent noise current, computed at $\mathcal{E}_N = 2.53 \times 10^{-11}$ for the amplification $G = 10^6$, are tabulated in the following chart on the basis of the irradiances computed for a temperature of 6000° K.

Available Anode Current I_{az} at $B/B_{\odot} = 1 \times 10^{-13}$
Signal to Noise Ratio for T_p = 45%
V.S. Wavelength λ

Wavelength λ A°	Irradiance $[E_e]$ 6000 K	Anode Current I_{az} Amp	Ratio $I_{az}/\mathcal{E}_N$
6500	180 ergs	3.94×10^{-11} Amp.	1.55
6000	196	3.22×10^{-10}	12.72
5500	209	1.225×10^{-9}	48.4
5300	213	1.70×10^{-9}	67.2
5000	217	1.855×10^{-9}	73.4
4500	201	1.95×10^{-9}	77.2
4000	198	1.745×10^{-9}	68.8
3500	164	0.967×10^{-9}	38.2

Considering that a signal to noise ratio of 1/1 can be managed through appropriate electronics, the feasibility is acquired at least for the range of 6000 A^{o} down to 3500 A^{o}.

However, the equivalent dark current input being of some 2.03 x 10^{-9} amper, the light energy input must be 100% modulated. The obtained values are typical and can be improved by means of selection of the photomultiplier for both cathode sensitivity and noise level. In what extent, this cannot be reported since no exacting measurements have been secured as yet. Nevertheless, though somewhat conservative, the indicated parameters satisfy the requirements of the problem.

If there is no major objection from the scientific point of view, it is suggested that the band pass of $\Delta\lambda$ = 100 A^{o} be not decreased or at least not below 50 A^{o}. For reliability, it is preferable to obtain the largest possible signal to noise ratio and thus avoid complexity of the electronics.

Naturally in the range of elongations from 2^{o} to some 12^{o} the signal would have to be attenuated since the brightness of the zodiacal light reaches some 5 x 10^{-10}. In that region one could take advantage of this fact to reduce the band pass to rather small values and achieve therefore a better discrimination in light scattering functions.

Chapter X

BASIC INSTRUMENTATION FOR THE MEASUREMENT OF THE BRIGHTNESS and POLARIZATION OF THE ZODIACAL LIGHT

The measurement of the brightness and of the degree of polarization of the zodiacal light dictates a minimum requirement of related instrumentation. This instrumentation, carried by an O.S.O. vehicle must include means for automatic control and reference while incorporating safety features which would not be necessary for ground installation. It is certainly admissible that the instrumentation can be monitored and commanded, at least within certain limits, through the telemetry of the vehicle. But, monitoring and command should entail only a fraction of the available time while being compatible with the over all capability of the system. At that stage, the feasibility study has to be limited to the basic requirement for the instrumentation, keeping in line with achievable minima in power requirement, weight and volume. Final details in design and engineering will have to be studied separately.

Basically the instrumentation can be divided into several major groups.

Instrumental optics

Light energy transducer circuit

Electronics and cycling

Data handling and transmission

Each group can be and will have to be divided in subgroups or sub-assemblies to study their relative liaisons and dependencies.

Instrumental Optics

In spite of the fact that the instrumentation will operate above the earth's atmosphere, the value of the relative brightness $B/\bar{B}_{\odot}$ of the zodiacal light falling as low as 1×10^{-13} at an elongation of some 100^{o}, very little irradiant energy per unit area is available. A collector of energy is mandatory. Collectors of light energy could be, a priori, of the converging type, either catadioptric or dioptric, focusing all the light energy into an extremely small circle of confusion. But, the large angles of convergence, for the marginal rays, found in those systems are readily uncompatible with the angular acceptance of polarizing systems which is generally limited to a few degrees. The study of the optical collectors indicates that, in that case, the collector must be an afocal telescope system for over all compatibility.

The afocal telescopes are of two kinds. The positive-positive system and the positive-negative one. Both systems having a zero convergence transform an entering collimated beam into an emerging one of smaller diameter. The ratio of the diameters of the collimated beams is strictly defined by the ratio f'/f of the focal length of the secondary or back objective f' onto the focal length f of the primary or front objective.

$$D' = \frac{f'}{f} D \qquad \text{X-1}$$

However the types of afocal telescope differ from each other in terms of their over all length L. For the positive-positive system the over all length L_p is given by the sum of the focal distances of the objectives

$$L_p = f + f' \qquad \text{X-2}$$

while for the positive-negative one, the over all length L_n reduces to the difference of the focal distances, yielding

$$L_n = f - f' \qquad \text{X-3}$$

The positive-negative afocal telescope would be more suitable since its relative aspect ratio

$$\rho = \frac{L_n}{L_p} = \frac{f - f'}{f + f'} \qquad \text{X-4}$$

is the smallest one.

However the choice, here, is conditioned by the eventuality of a direct or near sighting into the sun. For that reason one must elect the positive-positive system. A well defined, real field stop diaphragm d_f can be positioned at the accessible focal plane of the primary objective. This diaphragm d_f also defines the angular acceptance U of the system and controls the degree of collimation U' of the emerging beam through the existing relationships

$$\tan U' = \frac{d_f}{f'} \qquad \text{X - 5}$$

$$\tan U = \frac{d_f}{f} \qquad \text{X - 6}$$

and for small angles

$$U' \simeq \frac{f}{f'} \times U \qquad \text{X - 7}$$

The angular fields of views U' and U have already been discussed in terms of requirements for light energy to secure an adequate and safe level of signal to noise ratio. Thus the tentative design revolves around the following dimensions:

U	=	angular field of view	$\simeq$	2 degrees
D	=	clear equivalent diameter	$\simeq$	6 cms
U'	=	angular collimation	$\simeq$	6 degrees
D'	=	clear equivalent diameter	$\simeq$	2 cms
f'/f	=	focal length ratio $= D'/D$	=	1/3

Those dimensions lead to a practical collecting coefficient S of some

$$S' = 398 \ cm^2 \times d^{\circ 2}$$

as already established and which has been found sufficient.

The exact focal lengths f and consequently f' are tentatively indicated at

f = focal length of primary objective $\cong$ 10 cms

f' = focal length of secondary objective $\cong$ 3.33 cms

u = angular convergence of marginal rays $\cong$ $16^{\circ}\ 40'$ leading to

to a practical over all length of the afocal telescope of:

$$L_p \cong 13.3 \text{ cms.}$$

No final decision can be taken at this point in view of the fact that the tolerable spherical and chromatic aberrations of the system will have to be exactly evaluated. Spherical aberrations, on axis, vary with the square of the diameter of the objective while the axial chromatic aberrations vary with the focal length. Only an optimum compromise can be achieved. However the ratio of the dimensions indicated for f and f' is in line with the established behavior of the afocal part of zoom lenses. It could be possible to reach a 1/3.5 ratio but at the expenses of serious optical complications which would entail a larger loss of light by reflection losses.

As to the nature of both primary and secondary objectives, they can be tentatively specified in view of the environmental conditions which forbid the use of fogging refracting materials and cementing.

The primary objective would incorporate two elements: a positive element working at the minimum of spherical aberration followed by a stigmatic positive meniscus which can be designed to correct, in part, some of the spherical aberrations of the front element. This compromise results in some losses of the stigmaticity of the meniscus.

Suprasil I - grade A - pure silica glass, answers fully the non fogging requirement for the refracting material and should be elected for the two components of the front objective.

FIGURE X - 1

MEASUREMENT OF THE BRIGHTNESS AND OF THE POLARIZATION OF THE ZODIACAL LIGHT
DIAGRAM OF THE INSTRUMENTATION

An afocal telescope including a primary positive objective and a secondary objective collects the irradiant energy and collimates the light beam.

df - field stop diaphragm defining the angular field of view U

Q_{r1}, Q_{r2}, Q'_{r1} and Q'_{r2} movable quartz plates rotators

MF - Monochromatic filters

Q/4 Quarter wave plate

D.C.P. Rouy double cut prism

Bg Black glass reflected light trap

K_r Kerr cell

S.C.P. Rouy single cut prism

Stop Stop diaphragm

P.M. Photo multiplier

The input of solar energy for reference is injected, on time sharing basis, through a side channel inclined at 16 1/2 ° from the normal to the telescope axis. The line of sight through the sun is tentatively phased 90° ahead of the telescope and at the elongation ϵ of the scan direction. The channel incorporates a light energy attenuator, not shown, and appropriate mirrors arrangement M to fold the system.

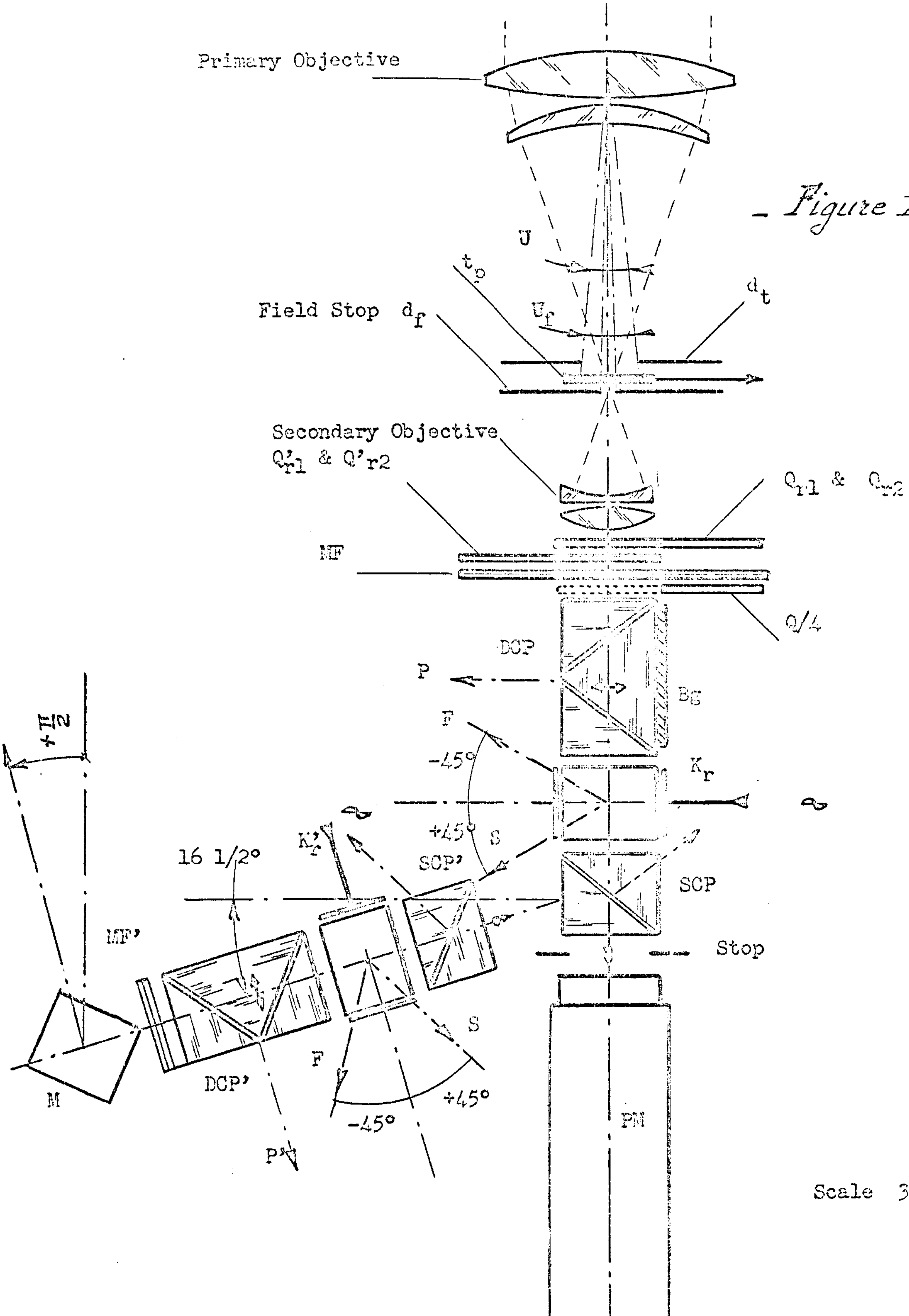
MF' Monochromatic filter

DCP' Rouy double cut polarizing prism

K'_r Kerr cell

SCP' Rouy single cut polarizing prism

- Figure X-1 -



Scale 3/5 Approx.

Measurement of the Brightness and of the Polarization of the Zodiacal Light

Diagram of the Instrumentation

The secondary or back objective being of smaller diameter allows for correction in spherical and chromatic aberrations, at least in part. Made of an uncemented doublet, including one positive and one negative elements, it will have to be over corrected in spherical and chromatic aberrations. The exact matching of the primary and secondary caustics cannot be expected. The choice of refracting materials is unquestionably Suprasil I, pure silica glass, for the positive element and tentatively saphire for the negative one. Since the secondary element is at least partially shielded from direct radiation, saphire could be replaced by some proven optical glass, if the rate of fogging is deemed compatible with the duration of the experimentation.

Ahead of the field diaphragm d_f, which defines the angle of field of view, and facing the front objective, one can locate a ring shaped light transducer t_p defining, through its free outside diameter d_t, a fail-safe angular field of view U_f of some six degrees or more. The output current of this transducer would be used, in case of excessive light energy due to accidental sighting into or near the sun, as a secondary fail-safe command. It can be either a barrier layer type photocell or a solid state photo conductive cell which, then, could be used for measurement of white light integrated energy in the direction of sight.

Thus the described afocal telescope provides for a flux of zodiacal light energy which, for a monochromatic filter having an equivalent band pass $\Delta\lambda$ of some 100 angstroms, centered for instance at 5300 A^{o}, gives access, through the whole integrated optical system, to a typical anode current output of

$$I_a \simeq 1.70 \times 10^{-9} \text{ amp.}$$

This anode current I_a appears to be in the ratio of some 67/1 with respect to the typical equivalent noise current of $\mathcal{E}_N = 2.53 \times 10^{-11}$ amper of the tentatively elected E.M.R. 541A-01-14 photomultiplier.

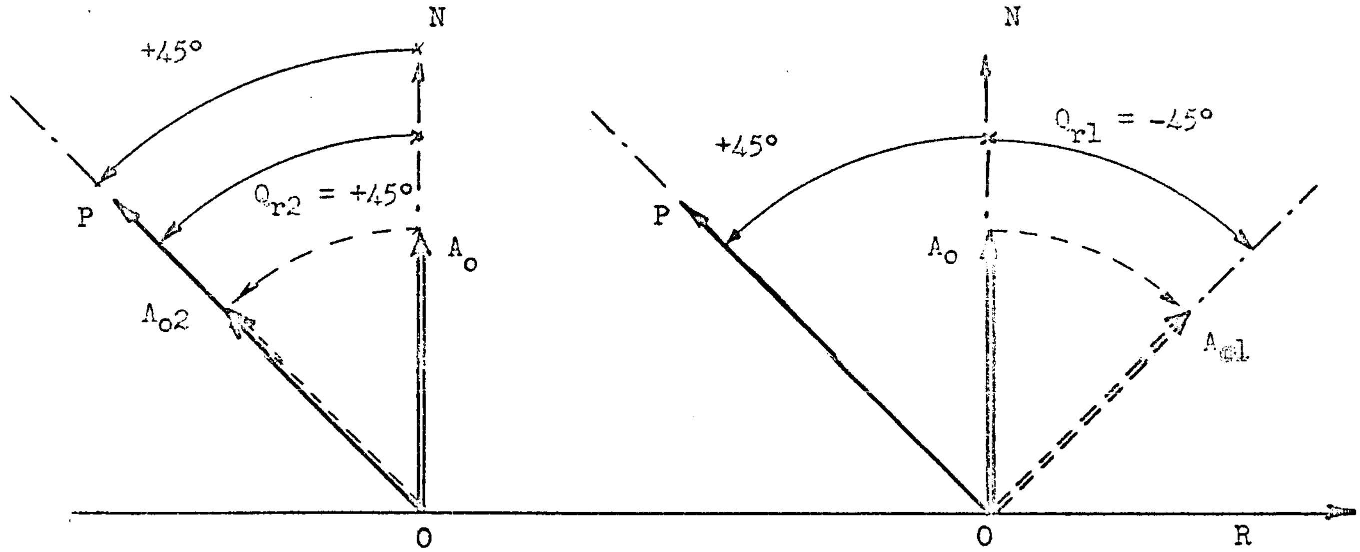
FIGURE X - 2 CASE I

The vectorial diagram shows the successive angular rotations of the amplitude vector A_o of the plane polarized light caused by the introduction of the rotators Q_1 or Q_2 to bring the vector A_o either in the position A_{o1} normal to the analyser axis OP or into the position A_{o2} parallel to it.

FIGURE X - 3 CASE II

Vectorial diagram showing the effect of the introduction of an auxiliary rotator $Qr_{\frac{1}{2}}$ causing an additive rotation of $+45^o$ of the direction of the amplitude vector A_o of the plane polarized light. The final plane polarized amplitude directions A'_{o2} and A'_{o1} make either the angle $+45^o + \Delta\alpha$ or the angle $- 45^o + \Delta\alpha$ with respect to the direction OP of the analyser axis giving access to the angular deviation $\Delta\alpha$

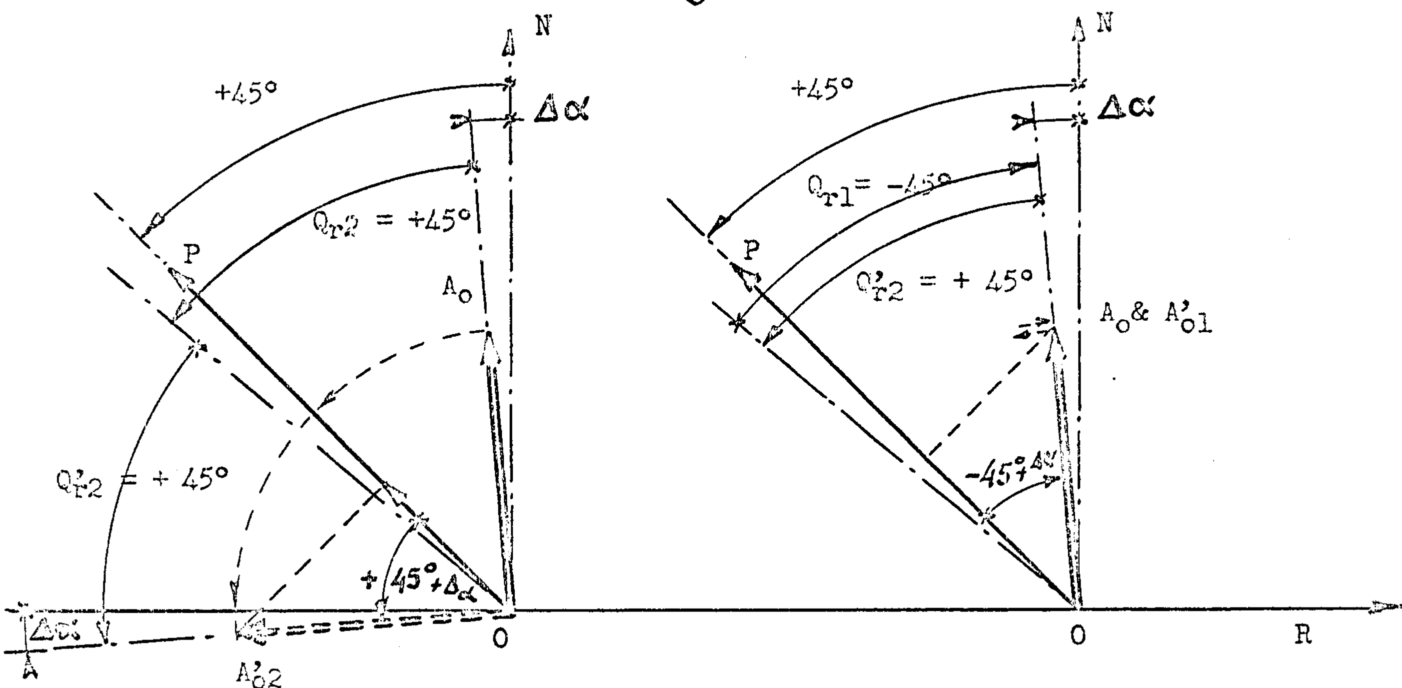
– *Figure X-2* –



Case I Plane Polarized Amplitude A_o

– A_{o2} Parallel to P – – A_{o1} Normal to P –

– *Figure X-3* –



Case II Plane Polarized Amplitude A_o

A'_{o2} at +45° + $\Delta\alpha$ from P A'_{o1} at -45° + $\Delta\alpha$ from P

The collimated light beam exiting from the secondary objective, in its progression toward the light transducer, passes first through a quartz crystal plate rotator Q_{r2} causing an angular rotation θ_2 of the direction of the plane polarized light, for instance $\theta_2 = + 45^\circ$. This rotator can be moved outside the collimated beam and replaced by the adjacent quartz crystal plate rotator Q_{r1} . This one will rotate the direction of the plane polarized light in a direction opposite to the previous one but equal in magnitude.

$$\theta_1 = -\theta_2 = -45^\circ$$

This arrangement permits the examination and measurement of the zodiacal light for its degree of polarization and brightness in accordance with the analytical development of the difference to the sum ratio. It is the minimum requirement since the rotation of the main analyzer onto the two preferred directions, normal and parallel to the ray issued from the sun and intersecting the line of sight, appears as not quite practical for several reasons. Considering the weight of those quartz crystal plates, a few grams, is already a very favorable factor in their favor.

The use of rotators for shifting the direction of the plane polarized amplitude entails the recording and integrating of several consecutive cycles of the same kind. It is deemed favorable in the over all picture since by this process, equivalent to a low band pass system one can reduce appreciably the noise level.

If more reliable data are to be obtained through shifting of the two axes of reference P_2 and P_1 by the angular distance $\alpha' = \pi/4$, as indicated in the analytical study, then those rotators must be either preceeded or followed by two other quartz crystal plate rotators Q'_{r1} , and Q'_{r2} causing the rotations $+\pi/4$ and $-\pi/4$. They are equal to the previous ones.

Thus with the rotators Q_r , the plane polarized light is entering the analyzer with the direction of its amplitude either parallel or normal to the axis of the analyzer. On the contrary, with the combination of the added rotators Q'_r the plane polarized light enters the analyzer with its amplitude at the angular distance $\pi/4 + \Delta\alpha$ or $\pi/4 - \Delta\alpha$. This last measurement provides the exact determination of the possible deviation $\Delta\alpha$ and also permits to introduce corrective terms, as defined in the analysis, into the value of the degree of polarization p_o .

Before entering the analyzer, the light beam traverses a monochromatic filter MF, interference type, having its peak of transmission centered onto the elected wave length and having a band pass of some 100 angstroms. The substrate of the monochromatic filter, or filters, should be preferentially Suprasil I pure silica glass. Several of those filters can be arranged into a proper carriage, (wheel) ~~which~~, to be introduced electively onto the beam, upon command. Eventually the monochromatic filters could be replaced in a more sophisticated design by a miniaturized and simplified grating monochromator providing that it be placed after the analyzer system. This position would have to be observed to eliminate the component of plane polarization, normal to the exit slit, characteristic of any monochromator behavior.

Considering the fact that the light beam delivered by the afocal collecting telescope is already collimated the realization of the monochromator would entail the addition of a grating, about 1 x 1 square inch, a small off axis parabolic mirror, ~~to~~ 1 inch in diameter and one exit slit or exit pupil of the filar optics type. The definition of the band pass would have to be computed and compared against the achievable spectral transmission of an interference monochromatic filter.

The monochromator would offer a better investigation of the influence of the wave length upon the scattering. An aspect of real importance.

At the emergence of the monochromatic filter MF, the light beam enters the analyzer proper DCP as shown in the diagram of the instrument. For this analyzer, whose axis of polarization P is preferably located at 45° from the normal N to the radius R issued from the sun and intersecting the line of sight, the choice of a Rouy's double cut prism is favored. Its over all transmission exceeds 45%, though somewhat lower than the transmission obtainable with the single cut Rouy's prism, it is about double the value of the transmission characteristic of the Glan's prism.

Moreover, contrary to the other types of polarizing prisms its field of view, some 8 1/2°, is positively defined and limited by total rejection for rays whose angle of incidence exceeds 4 1/4° in absolute value from the axis of the prism. The light beam exiting from such prism is always 100% polarized. For further precaution a plate of black glass Bg is maintained in close contact with the long side of the prism. Its purpose is to lower the light energy level of the light reflected toward the telescope by reflection on the side. It is not a necessary procedure but an added safety feature.

Following the analyzer prism DCP, a Kerr's cell Kc is placed onto the beam with its fast F and slow S axes orientated to form angles of +45° and -45° respectively with the axis of the polarizing prism. This Kerr's cell has a duality of purpose and governs the functioning of the instrument.

Upon its excitation, at the adequate field gradient, the Kerr's cell transforms the plane polarized light into circular polarized light energy which can then traverse a polarizing prism SCP, placed after the cell, at 50% transmission irrelevant of its angular orientation. On the contrary, when the exciting electrostatic field is not applied, the plane polarized light remains unaffected in both amplitude and nature and enters the second polarizing prism SCP. This prism, which can be a Rouy's single cut prism on account of its higher transmission, is orientated with its axis normal to the direction of the axis of the double cut prism to secure total extinction of the plane polarized light.

Thus, when the Kerr's cell is not excited, no light can reach the photomultiplier. On the contrary, the light energy reaches the photomultiplier when the Kerr's cell is excited.

The first purpose of the Kerr's cell is to introduce a 100% modulation of the light energy reaching the phototransducer as imposed by the dark current computed at $\mathcal{E}_D = 2.03 \times 10^{-9}$ amper. At what frequency the Kerr's cell should be modulated is presently the matter of an analytical treatment in view of the possibility of attenuating the noise caused by a star crossing. Unquestionably, the frequency of modulation should not be chosen smaller than that equivalent to an angle of scan of 3 degrees. In the case of the O.S.O. vehicle, rotating at some 1/2 RPS, the frequency should not be lower than

$$\nu \geq 60 \text{ cycles per second.}$$

The second purpose of the Kerr's cell is to protect the light transducer from excessive irradiation. Indeed, the field of excitation can be collapsed instantaneously when a properly angularly located sensor sends a signal of proper strength into the power source of the modulator. For instance, the suggested annular sensor t_p, placed ahead of the field diaphragm d_f can be at least one of the safety sensors. It is deemed safer to incorporate a second command acting independently and preceeding, in angular relationship, the afocal telescope. This sensor could impose a time constant, for the recovery of the modulation, of such magnitude that the protection would be 100% effective in all eventuality.

There is also a phase of the scanning cycle which is worth specific attention. For approximatively half of the period of revolution of the vehicle around the earth, the telescope will be directed toward the earth surface. First looking at the upper layers of the atmosphere then later at the dark surface. For a short period of time the telescope will be directed along a

FIGURE X - 4 a

The field of view limited to some 8 1/2° for both Glan and Rouy single cut prisms is bounded at the right by total rejection of light energy and at the left by admission of unpolarized light

FIGURE X - 4 b

The field of view, limited to some 8 1/2° for a Rouy double cut prism, is bounded on each side by total rejection of light energy. The prism can pass only the extra ordinary polarized light energy.

FIGURE X - 5

The light energy of reference E_r, obtained from direct sighting into the sun through a referencing channel incorporating a calibrated attenuator, is injected into the system at half angular distance between the peaks of the zodiacal light occuring at the elongations $\mathcal{E}_1$ and $\mathcal{E}_2$

Polarising Prisms

Aspects & Limits of field of view

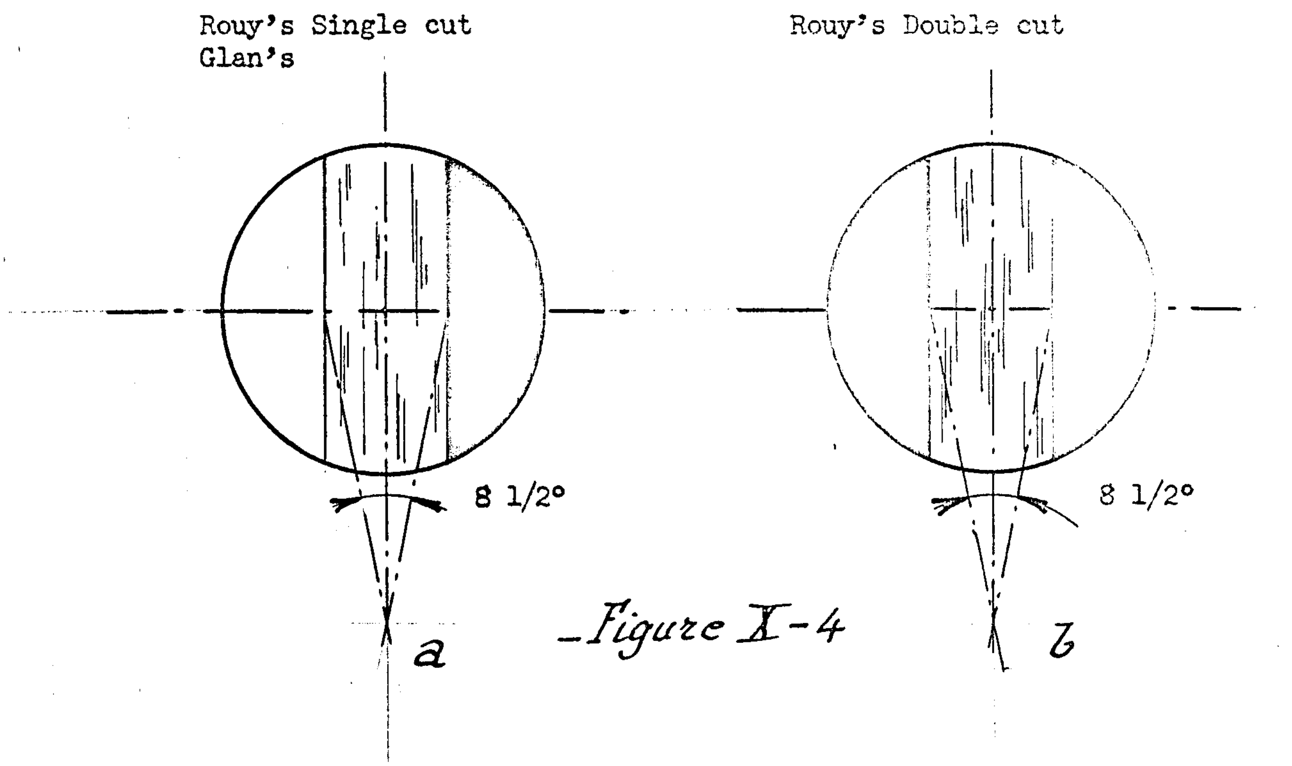
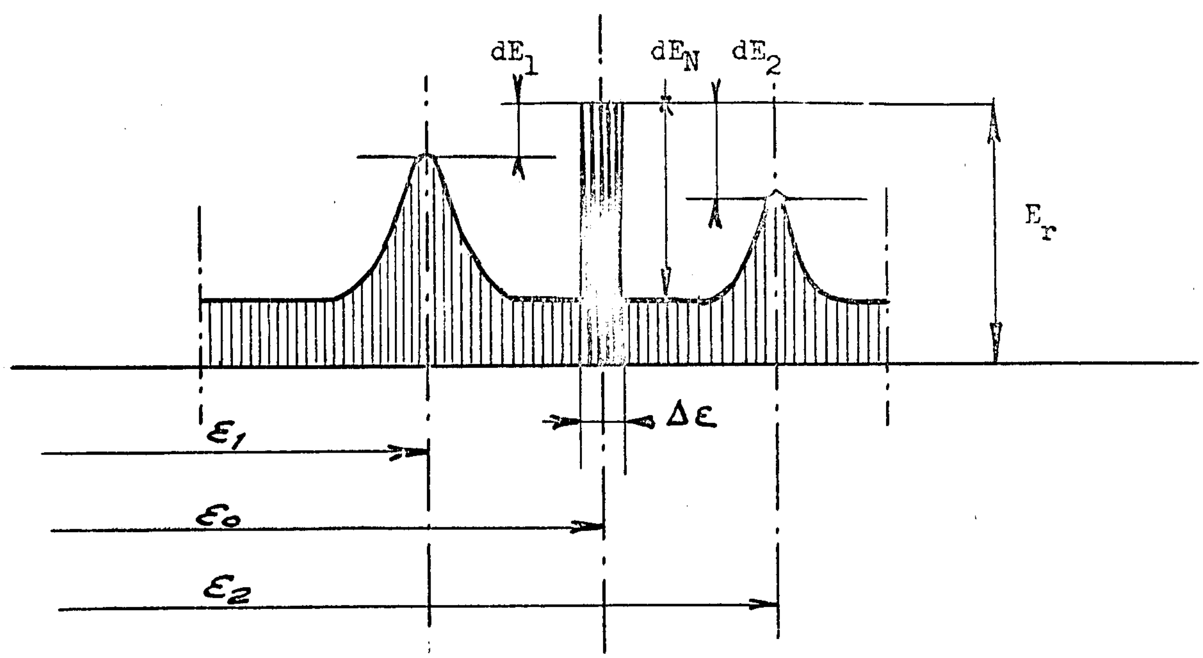


_Figure X-4

_Figure X-5



Brightness of the Zodiacal Light
Injection of Reference Light Energy E_r

tangent to the upper layers of the atmosphere. It is conceivable that measurements of radiated energies in those areas would have significance. This question has not been examined as yet. But the basic instrumentation could be brought to bear into that problem using the fail-safe circuitry to control the procedure. Indeed, it is possible to command and control the strength of the modulating electrostatic field to govern the light level reaching the transducer and this within the limits of 0 to 100%.

A reference of light energy level is of practical necessity. If possible the reference should be taken with respect to the sun's brightness. At least several methods of referencing can be designed. However special attention is directed to the fact that reliability is of prime importance. Along this line it is highly desirable that the light transducer which is detecting the brightness of the zodiacal light be the one looking at the reference source. This is a rather serious problem. The light energy for the reference must have, at the photo-cathode, precisely the same physical nature as the light emerging from the last polarizing prism SCP, the same density distribution function and the same angle of incidence.

A practical solution is proposed which satisfies basically the requirements.

For the light transducer PM to look into the reference source, use is made of the air inter face reflecting back surface of the prism SCP. One can inject the referencing light energy at an angle of some $16^{\circ}\ 30'$ from the normal to the side surface. This referencing beam, being plane polarized with the direction of its amplitude parallel to the reflecting surface, that is, parallel to that of the plane polarized energy which traverses normally the system, becomes reflected toward the sensor and precisely along the main axis. The reflecting power τ is rather high owing to the angle of incidence at the air space surface for one part and to the fact that the amplitude is parallel to the reflecting surface on the second part. The reflecting power r has been evaluated at 47.3% for a wave length of 5300 A°. Thus, the referencing beam being collimated at the same value U', as prevailing along the main axis, is also of the same nature as that of the normal beam, that is plane polarized, with its amplitude normal to the direction of the amplitude of the normal beam at its emergence from the prism SCP. To bring the referencing amplitude to be parallel to that of the main beam then it suffices to orientate the axis of the prism SCP' to be parallel to the plane normal to the diagonal cut of the prism SCP. In that relative angular position, a far lower coefficient of reflection is obtained.

The referencing beam traverses, before reaching the prism SCP, an equivalent optical circuit including a double cut prism DCP', a Kerr's cell K_r' and a single cut prism SCP'. This part of the optical circuit of reference will follow a light collector pointed toward the sun and the appropriate attenuator. But light collector and attenuator become rather small in view of the considerable relative brightness of the source. Treatment of this part by Fiber Optics appears as one of the simplest and most reliable solutions.

In its functioning, the reference circuit, whose line of sight is phased at half the angular distance between two consecutive peaks of the zodiacal light, first commands the collapsing to zero of the energizing field of the Kerr's cell K_r on the telescope axis and opens the modulated field on the Kerr's cell K_r'. The operation must be gated through a derivation in the referencing circuit. Thus the modulated energy of the reference source enters the light transducer and the amplified output stored as reference voltage in an electrometric tank circuit. Whence the angular scan, a few degrees, through the reference channel is completed, the modulation is cut off on the Kerr's cell K_r' and restored on the main Kerr's cell K_r.

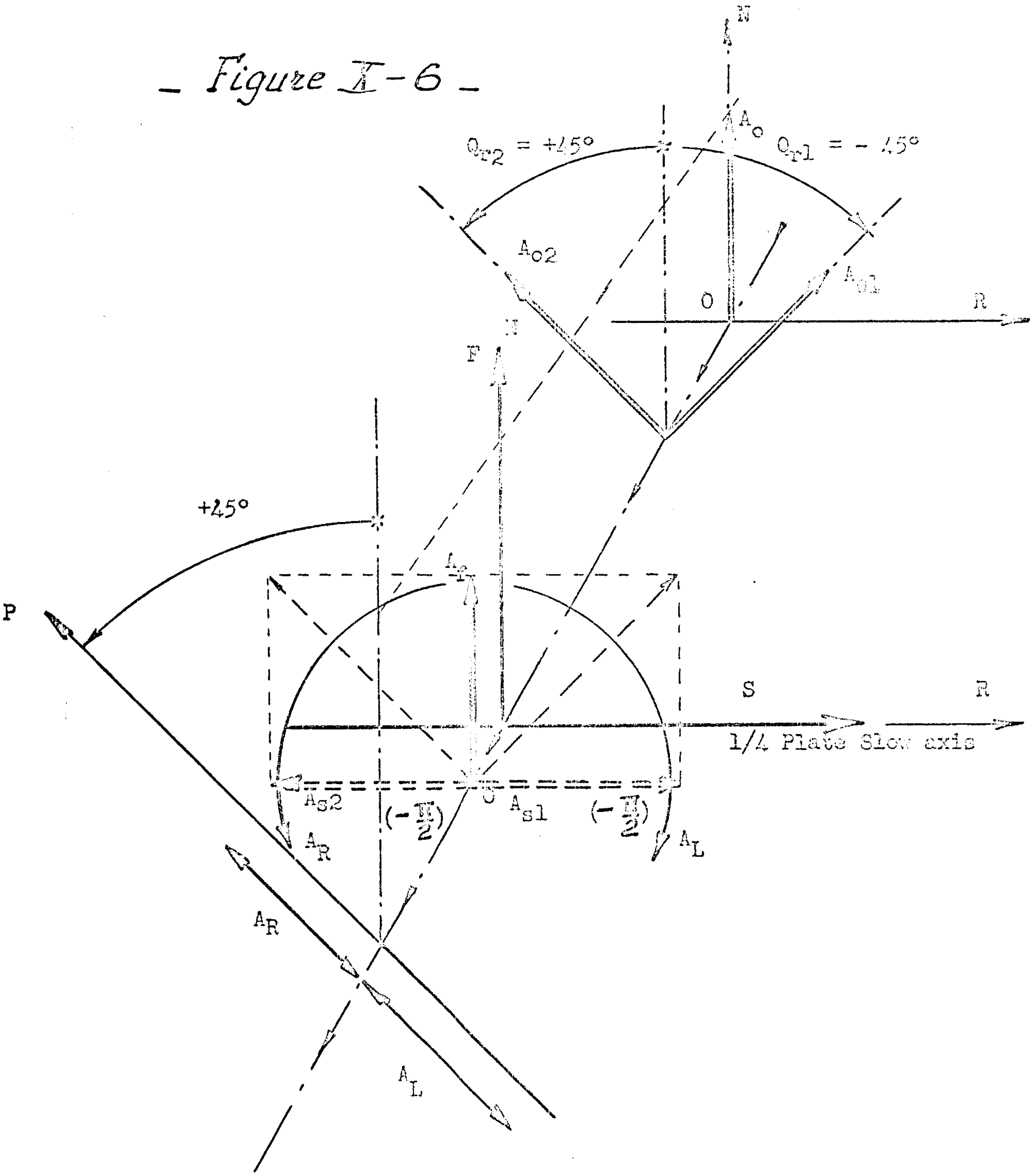
FIGURE X - 6

DETERMINATION OF THE ELLIPTICITY OF THE ZODIACAL LIGHT

The plane polarized amplitude A_o, normal to the radius OR passing through the sun, is rotated either by +45° into A_{o2} by the rotator Q_{r2} or by - 45° into the direction A_{o1} before traversing a quarter wave plate whose fast axis F is set parallel to the direction A_o. At the emergence from the quarter wave plate, the plane polarized light amplitude A_o is transformed either into right circular polarized light $A_R = (A_f, A_{s2})$ or left circular polarized light $A_L = (A_f, A_{s1})$ before entering the analyser whose axis P makes an angle +45° with the normal to the ray OR.

The difference $A_R - A_L$ between the amplitudes of right and left circular polarized energies emerging from the analyser measures the ellipticity since those components A_R and A_L are added or subtracted from the circular polarized energy component.

_ Figure X-6 _



Determination of the Ellipticity of the Zodiacal Light

Finally, considering the measurement of the ellipticity, of the zodiacal light, in accordance with the analytical expose, it suffices to introduce the quartz crystal quarter wave plate Q/4 onto the beam and ahead of the main analyzer DCP. The logistics of the measurement are identical to the ones described for the measurement of the degree of polarization and relative brightness of the zodiacal light.

Barring limitations imposed by the O.S.O. vehicle, the feasibility appears satisfactory. The electronics are not discused here since they are available in classical subminiaturized form entirely compatible with the output of the light detector. A final analysis will have to be made with respect to the existing circuitry of the vehicle.